Update of the calculation of metabolizable energy requirements for pigs in the German agricultural emission inventory

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Abstract

The calculation of the energy requirements of pigs used in the German agricultural emission inventory is updated on the basis of the recommendations worked out by the German Society of Nutrition Physiology (GfE). At the same time the numeric solutions hitherto used in the calculations of energy requirements of weaners and fattening pigs in the emission inventories are replaced by analytical solutions. This results in simplifications in the calculation procedures.

In the inventory this leads to reduced energy requirements for fattening pigs. However, the energy requirements for sows, weaners and boars increase.

Keywords: pigs, energy requirements, model, emission inventory

Zusammenfassung

Aktualisierung der Berechnung des Bedarfs an umsetzbarer Energie von Schweinen im deutschen landwirtschaftlichen Emissionsinventar


Im Inventar ergeben sich daraus für Mastschweine eine Verringerung des Energiebedarfs, für Sauen, Aufzuchtferkel und Zuchteber jedoch Erhöhungen.

Schlüsselwörter: Schweine, Energiebedarf, Modell, Emissionsinventar

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1 Introduction

Agricultural production involves the emission of trace gases whose effects on the atmosphere and the environment may be adverse. Attempts to reduce these emissions presuppose their detailed quantification in a way that allows for the identification and evaluation of the relevant processes. As emission reduction goals are being discussed in international boards, the respective parties use standard approaches to describe and quantify emissions. In order to avoid distortion of the markets, international guidance documents were established that allow the assessment of emissions and the description of emission explaining variables in a consistent way (greenhouse gases: IPCC, 2006; air pollutants: EMEP/EEA, 2009).

Emissions have to be reported as consistent time series for the period from 1990 onwards. For key sources, calculations and reporting have to be detailed. Hence fattening pigs will be treated more sophisticated than boars.

At present, an attempt is being made to better describe emissions from pig production, including the description of feed composition (Dämmgen et al., 2011), updating emission factors (Dämmgen et al., 2010a) and technical measures to reduce emissions from houses (Dämmgen et al., 2010b).

All emission rates are related to excretion rates. These are depending on feed intake rates which are governed by the energy and nutrient requirements of the respective animals.

This work describes in detail the assessment of the energy requirements in pig production. It is restricted to the fraction of energy that can be metabolized (metabolizable energy, ME).

Hitherto, the assessment of energy requirements in pig production in the German agricultural emission inventory was based on data provided in GfE (1987). Meanwhile, the respective data were updated and the methods improved (GfE, 2006). The comprehensive revision of the description of emissions from pig production makes use of this new edition.

To a large extent, the literature used in GfE (2006) was published during the time span underlying the emission inventory (the years from 1990 onwards). Hence, it is assumed that the data and methods compiled in GfE (2006) are representative of the past two decades and that the calculation procedures derived from GfE (2006) are applicable to all the years since 1990.

This work applies and transforms the relations provided in GfE (2006) so that they meet the needs of the agricultural emission inventory.

2 Definition of pig subcategories

The assessment of energy requirements is carried out for animal populations with similar feeding. However, official German animal survey data rely on a differentiation of animals according to weights. Hence, the survey data have to be transformed to fit into energy requirement calculation routines. Table 1 compares the categories used in the official German survey and in the German emission inventory (see Haenel et al., 2010).

Table 1:
Pigs, categorization and characterization of subcategories

<table>
<thead>
<tr>
<th>descriptor in the official survey</th>
<th>type</th>
<th>subcategory in the emission inventory</th>
<th>weight 1</th>
<th>weight 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>piglets</td>
<td>---</td>
<td>suckling-pigs</td>
<td>1.5</td>
<td>8</td>
</tr>
<tr>
<td>young pigs lighter than 50 kg live weight</td>
<td>weaners</td>
<td></td>
<td>variable $w_{fa, ws}$</td>
<td></td>
</tr>
<tr>
<td>fattening pigs 50 to 80 kg</td>
<td>fp</td>
<td>fattening pigs</td>
<td>variable $w_{fa, vs}$</td>
<td></td>
</tr>
<tr>
<td>fattening pigs 80 to 110 kg</td>
<td></td>
<td></td>
<td>variable $w_{fa, vs}$</td>
<td></td>
</tr>
<tr>
<td>fattening pigs heavier than 110 kg</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>young sows gestating</td>
<td>so</td>
<td>sows</td>
<td>mean weight: 220</td>
<td></td>
</tr>
<tr>
<td>young sows not gestating</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>other sows gestating</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>other sows not gestating</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>boars</td>
<td>bo</td>
<td>breeding boars</td>
<td>mean weight: 200</td>
<td></td>
</tr>
</tbody>
</table>

* type: code used to describe the animal categories in the German inventory
* weight 1: weight at the beginning of the respective period, weight 2: weight at the end of the respective period; $w_{fa, ws}$: variable final weight.
* suckling-pigs do not constitute a separate animal category. They are accounted for within the calculations of energy and feed requirements of the sows (see Haenel et al., 2011)
The subcategory “sows” covers all subcategories of sows for breeding irrespective of their age and weight. The calculation of energy and feed requirements includes the suckling-pigs as they do not constitute a separate animal category in the inventory (see Haenel et al., 2011).

Weaners are young pigs between weaning and the beginning of fattening. In the emission inventory, this lifespan is assumed to begin at an animal weight of about 8 kg animal$^{-1}$. It ends with weights ranging from 25 to approx. 30 kg animal$^{-1}$ (depending on year and region). Data for $w_{\text{ins,fp}}$ are available for each year and district.

Fattening pigs are all pigs in the final fattening stages, i.e. with weights above about 25 to 30 kg animal$^{-1}$ till slaughtering (105 to 120 kg animal$^{-1}$ live weight). Again, data for $w_{\text{ins,fp}}$ are available for each year and district.

For sows a mean weight of 220 kg animal$^{-1}$ is estimated as discussed below.

For breeding boars, a mean live weight of 200 kg animal$^{-1}$ is assumed (expert judgment E. Schulz).

The categories used in the inventory are described in the subsequent chapters. The variable animal weights used in Table 1 for the inventory category “weaners” and “fattening pigs” are obtained from official statistics. The weight data for piglets are based on expert judgment, while the mean weights of sows and boars will be discussed in the respective chapters.

3 Sows

3.1 Total energy requirements during one reproduction cycle

The total metabolic energy (ME) requirements of a sow during one round, i.e. during one reproduction cycle, is given by Equation (1). It reflects that the reproduction cycle comprises two gestation phases, the lactation phase and the time span between weaning and covering.

\[ \Sigma \text{ME}_{\text{sow}} = \Sigma \text{ME}_{\text{gest,1}} + \Sigma \text{ME}_{\text{gest,2}} + \Sigma \text{ME}_{\text{lact}} + \Sigma \text{ME}_{\text{wmc}} \]  

where

- $\Sigma \text{ME}_{\text{sow}}$ total metabolizable energy required for one round (MJ animal$^{-1}$ round$^{-1}$)
- $\Sigma \text{ME}_{\text{gest,1}}$ metabolizable energy required during gestation phase 1 (in MJ animal$^{-1}$ round$^{-1}$)
- $\Sigma \text{ME}_{\text{gest,2}}$ metabolizable energy required during gestation phase 2 (in MJ animal$^{-1}$ round$^{-1}$)
- $\Sigma \text{ME}_{\text{lact}}$ metabolizable energy required during lactation (in MJ animal$^{-1}$ round$^{-1}$)
- $\Sigma \text{ME}_{\text{wmc}}$ metabolizable energy required between weaning and covering (in MJ animal$^{-1}$ round$^{-1}$)

The phase-related energy requirements will be discussed in the subsequent chapters.

3.1.1 Energy requirements during gestation

The energy requirements for the development of the conception products, the uterus and the compensation of weight losses during the previous gestation have to be taken into account in addition to the energy requirements for maintenance (GfE, 2006, pg. 34f). The gestation phase can be subdivided into two separate phases with different means of daily ME requirements $\text{ME}_{\text{gest,1}}$ and $\text{ME}_{\text{gest,2}}$ (with $\eta$ the index of day).

GfE (2006), Table 4.13, provides $\text{ME}_{\text{gest,1}}$ and $\text{ME}_{\text{gest,2}}$ data depending on the ordinal number of pregnancy and the weight loss during lactation. As no official statistics are available on these criteria, the GfE data have been averaged with identical weights yielding $\text{ME}_{\text{gest,1}} = 33.3$ MJ d$^{-1}$ and $\text{ME}_{\text{gest,2}} = 40.7$ MJ d$^{-1}$ for the respective periods.

The ME data in Table 4.13 in GfE (2006) represent the energy requirements when the sow is expected to give birth to 13 piglets. At present, this piglets number per litter marks the upper limit in German piglet production. Hence, the use of $\text{ME}_{\text{gest,1}}$ and $\text{ME}_{\text{gest,2}}$ would avoid underestimation of the energy requirements used for the construction of emission time series.

3.1.2 Energy requirements during lactation

During the lactation period, energy is required for maintenance and milk production. Equation (2) defines the total daily energy requirements for the sow-piglets system as it is applied in the inventory.

\[ \text{ME}_{\text{lact, j}} = \text{ME}_{\text{m,sow, j}} + \eta_{\text{milk}} \cdot n_{\text{piglets, raised}} \]

where

- $\text{ME}_{\text{lact, j}}$ daily metabolizable energy required on day $j$ during lactation (in MJ animal$^{-1}$ d$^{-1}$)
- $j$ index of day
- $\text{ME}_{\text{m,sow, j}}$ daily metabolizable energy required for maintenance on day $j$ (in MJ animal$^{-1}$ d$^{-1}$, see below)
- $\eta_{\text{milk}}$ specific metabolizable energy required for milk production (in MJ piglet$^{-1}$ d$^{-1}$, see below)
- $n_{\text{piglets, raised}}$ number of piglets raised per sow and birth (in piglet, see below)

The total energy requirements for maintenance during a gestation phase are calculated from the daily requirements for maintenance which are given in GfE (2006), pg. 23, Equation 7:

\[ \text{ME}_{\text{m,sow, j}} = \eta_{\text{milk, m,sow}} \cdot W_{\text{ins, j}} \]
where

\[ ME_{\text{m, sow}, j} \] metabolizable energy required for maintenance for day \( j \) (in MJ animal\(^{-1} \) day\(^{-1} \))

\( j \) index of day

\( \eta_{\text{ME, m, sow}} \) specific metabolizable energy required for maintenance (\( \eta_{\text{ME, m, sow}} = 0.44 \) MJ per kg of metabolic weight, see footnote to Table 4.13 in GfE, 2006)

\( w_{\text{met}, j} \) metabolic weight on day \( j \) (in kg animal\(^{-1} \))

The metabolic weight, \( w_{\text{met}, j} \), is a function of animal weight \(^1\):

\[ w_{\text{met}, j} = w_{\text{unit}} \left( \frac{w_{\text{sow}, j}}{w_{\text{unit}}} \right)^{0.75} \] (4)

where

\( w_{\text{met}, j} \) metabolic weight on day \( j \) (in kg animal\(^{-1} \))

\( w_{\text{sow}, j} \) daily mean of live weight on day \( j \) (in kg animal\(^{-1} \), see below)

\( w_{\text{unit}} \) unit weight (\( w_{\text{unit}} = 1 \) kg animal\(^{-1} \))

In the inventory the daily mean of live weight, \( w_{\text{sow}, j} \), is replaced by the sow weight averaged over the entire reproduction cycle, see Chapter 3.3.

The specific metabolizable energy required for milk production, \( \eta_{\text{milk}} \), can be derived from the data provided in the footnotes to Table 4.17 in GfE (2006), pg. 81: production of milk per kg piglet weight gain (4.1 kg kg\(^{-1} \)), energy content of milk (5.0 MJ kg\(^{-1} \)), percentage of metabolizable energy used for milk production (70 %). These data lead to \( \eta_{\text{milk}} = 6.8 \) MJ piglet\(^{-1} \) day\(^{-1} \), when being combined with an average estimate for daily piglet weight gain of 232 g piglet\(^{-1} \) day\(^{-1} \) which is obtained from birth and weaning weights and the duration of the lactation phase (see Table 2).

The number of piglets per litter can be deduced from the number of piglets raised per sow and year and the duration of a production cycle (see Table 2).

\[ n_{\text{piglets, raised}} = n_{\text{piglets, year}} \cdot \frac{\tau_{\text{round}}}{\alpha} \] (5)

where

\( n_{\text{piglets, raised}} \) number of piglets raised per sow and birth (in piglet)

\( n_{\text{piglets, year}} \) number of piglets raised per sow and year birth (in piglet)

\( \tau_{\text{round}} \) duration of piglet production cycle (in d round\(^{-1} \))

\( \alpha \) time units conversion factor (\( \alpha = 365 \) d a\(^{-1} \))

Note that Equation (2) does not explicitly account for the possible gains of metabolizable energy by weight losses during lactation and/or additional piglet feed (GfE, 2006, pg. 38) as there are no official statistical data to describe these contributions. However, Equation (2) covers a wide range of ME requirements data provided in Table 4.17 in GfE (2006). Hence it is assumed that, on average, Equation (2) describes the German situation adequately.

Table 2 lists results of Equation (2) obtained for a range of piglet numbers.

### 3.1.3 Energy requirements between weaning and covering

For the period between weaning and covering, GfE (2006), pg. 72, assume the same total daily energy requirements as for advanced gestation.

\[ ME_{\text{wean, j}} = ME_{\text{gest, 2}} \] (6)

where

\( ME_{\text{wean, j}} \) daily metabolizable energy required during the phase "weaning to covering" (in MJ animal\(^{-1} \) day\(^{-1} \))

\( j \) index of day

\( ME_{\text{gest, 2}} \) daily metabolizable energy required during advanced gestation (in MJ animal\(^{-1} \) day\(^{-1} \)), see Chapter 3.1.1

### 3.2 Cumulative energy requirements and comparison with the former approach

Table 2 lists the daily requirements for the different phases (see Chapters 3.1.1 to 3.1.3) as well as the duration of the respective phases.

Table 2:

<table>
<thead>
<tr>
<th>phase</th>
<th>duration in d</th>
<th>number of piglets per sow per birth</th>
<th>energy requirements ***, revised approach in MJ animal(^{-1} ) day(^{-1} )</th>
<th>energy requirements, former approach * in MJ animal(^{-1} ) day(^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>gestation phase 1</td>
<td>84 *</td>
<td>33.3</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>phase 2</td>
<td>30 *</td>
<td>40.7</td>
<td>29</td>
<td>29</td>
</tr>
<tr>
<td>lactating</td>
<td>28 **</td>
<td>79.5</td>
<td>56</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>86.3</td>
<td>67</td>
<td>67</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>93.1</td>
<td>77</td>
<td>77</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>99.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>106.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>113.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>weaning to covering</td>
<td>27 *</td>
<td>40.7</td>
<td>29</td>
<td>29</td>
</tr>
</tbody>
</table>

* based on GfE (1987); see also Dämmgen et al. (2007); ** in connection with a weaning weight of 8 kg piglet\(^{-1} \) (expert judgment E. Schulz); *** based on GfE (2006)
Table 2 also allows for a comparison with the values hitherto used which were based on GfE (1987) (see Dammgen et al., 2007) and considerably fall below those obtained with the new approach.

The cumulative energy requirements, i.e. the total of energy required during one reproduction cycle, is provided in Table 3.

Table 3: Sows, total of energy required during one reproduction cycle ($\Sigma ME_{sw}$)

<table>
<thead>
<tr>
<th>number of piglets</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma ME_{sw}$ (in MJ animal$^{-1}$ round$^{-1}$)</td>
<td>7166</td>
<td>7331</td>
<td>7496</td>
<td>7661</td>
<td>7826</td>
<td>7992</td>
</tr>
</tbody>
</table>

3.3 Animal weights

3.3.1 Estimation of mean sow weight

In general, the live weight of a sow is not a constant during the reproduction cycle. The weight gain during gestation compensates the weight loss during lactation in the precedent reproduction cycle and comprises the weight gain of the conception products and the mammary gland as well as the weight gain of the sow itself. Table 4 shows typical weight change data (taken from Table 4.13 in GfE, 2006). A weight of 255 kg animal$^{-1}$ is considered a typical final live weight (DLG, 2008, pg. 13).

Table 4: Sows, animal weights ($w_{sw}$) and weight changes ($\Delta w_{sw}$) as a function of the number of gravidity ($n_g$). All weight data in kg animal$^{-1}$. (Source: GfE, 2006, Table 4.13.)

<table>
<thead>
<tr>
<th>$\Delta w_{sw}$ during lactation</th>
<th>$w_{sw}$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 $w_{sw}$ at covering</td>
<td>140</td>
<td>185</td>
<td>225</td>
<td>255</td>
<td></td>
</tr>
<tr>
<td>$\Delta w_{sw}$ during gravidity</td>
<td>70</td>
<td>65</td>
<td>55</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>-10 $w_{sw}$ at covering</td>
<td>140</td>
<td>185</td>
<td>225</td>
<td>255</td>
<td></td>
</tr>
<tr>
<td>$\Delta w_{sw}$ during gravidity</td>
<td>80</td>
<td>75</td>
<td>65</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>-20 $w_{sw}$ at covering</td>
<td>185</td>
<td>225</td>
<td>255</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta w_{sw}$ during gravidity</td>
<td>85</td>
<td>75</td>
<td>45</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

However, these weight changes cannot be accounted for in the inventory as the time span taken by one reproduction cycle is shorter than the inventory time step which is one year. Hence energy considerations for the inventory have to be based on a mean sow weight. Simply averaging the weights at covering as listed in Table 4 yields 2.1 10$^5$ kg animal$^{-1}$. However as this estimate may be biased by a relative under-representation of fully grown-up sows, the inventory assumes a mean sow weight of 2.2 10$^5$ kg animal$^{-1}$, which is the arithmetic mean of the weights at covering for the second and the fourth gestation. It ignores the lowest weight in order to roughly compensate for the under-represented highest weight.

3.3.2 Mean piglet weights at birth and weaning

For piglets, the inventory is based on a birth weight of 1.5 kg piglet$^{-1}$. This can be derived from feeding recommendations in DLG (2008), Table 4.1, where the piglet weight after the first week with a daily weight gain of 0.2 kg piglet$^{-1}$ d$^{-1}$ reaches 2.9 kg piglet$^{-1}$.

According to expert judgment (E. Schulz) the typical weaning weight is assumed to be 8 kg piglet$^{-1}$ in connection with a lactation phase duration of 28 days (see Table 2).

4 Weaners

For weaners, the metabolic energy requirements comprise the requirements for both maintenance and growth. They are calculated according to the recommendations in GfE (2006).

$$\Sigma ME_{we} = \sum_{j=1}^{k_{fn}} \tau_{day} \cdot ME_{m, we, j} + \sum_{j=1}^{k_{fn}} \tau_{day} \cdot ME_{p, we, j}$$

(7)

where

$\Sigma ME_{we}$ total metabolizable energy required (MJ animal$^{-1}$ round$^{-1}$)

$j$ index of day

$k_{fn}$ number of the final day of the weaners’ life span

$\tau_{day}$ unit time period ($\tau_{day} = 1$ d round$^{-1}$)

$ME_{m, we, j}$ metabolizable energy required for maintenance for day $j$ (in MJ animal$^{-1}$ d$^{-1}$)

$ME_{p, we, j}$ metabolizable energy required for growth for day $j$ (in MJ animal$^{-1}$ d$^{-1}$)

4.1 Daily energy requirements

Daily energy requirements for maintenance with weaners are defined as follows (cf. discussion Chapter 3.1.2).

$$ME_{m, we, j} = \eta_{ME, m, we} \cdot \frac{w_{we, j}}{w_{unit}}$$

(8)

where

$ME_{m, we, j}$ metabolizable energy required for maintenance for day $j$ (in MJ animal$^{-1}$ d$^{-1}$)

$j$ index of day

$\eta_{ME, m, we}$ specific metabolizable energy required for maintenance ($\eta_{ME, m, we} = 1.25\cdot0.44$ MJ kg$^{-1}$ d$^{-1}$, GfE, 2006, pg. 23)

$w_{we, j}$ mean weaner live weight on day $j$ (in kg animal$^{-1}$)

$w_{unit}$ unit weight ($w_{unit} = 1$ kg animal$^{-1}$)
The energy required daily for growth is calculated according to:

\[ ME_{g,\text{we}, j} = \eta_{ME,\text{g,we}, j} \cdot \left( \frac{\Delta w}{\Delta t} \right)_{\text{we},j} \]  
(9)

where

- \( ME_{g,\text{we}, j} \) metabolizable energy consumed for daily growth on day \( j \) (in MJ animal\(^{-1}\) d\(^{-1}\))
- \( j \) index of day
- \( \eta_{ME,\text{g,we}, j} \) specific metabolizable energy required for growth on day \( j \) (in MJ kg\(^{-1}\), see below)
- \( \left( \frac{\Delta w}{\Delta t} \right)_{\text{we},j} \) daily weight gain per animal and day \( j \) (in kg animal\(^{-1}\) d\(^{-1}\))

The specific metabolizable energy required for growth is a function of protein and fat gains, hence a function of age:

\[ \eta_{ME,\text{g,we}, j} = \frac{\alpha_{p,\text{we}} \cdot P_{\text{we}} + \alpha_{f,\text{we}} \cdot F_{\text{we}, j}}{k_{pf,\text{we}}} \]  
(10)

where

- \( \eta_{ME,\text{g,we}, j} \) day-dependent specific metabolizable energy required for growth (in MJ kg\(^{-1}\))
- \( j \) index of day
- \( \alpha_{p,\text{we}} \) energy content of protein (\( \alpha_{p,\text{we}} = 23.8 \) MJ kg\(^{-1}\), see GfE, 2006, pg. 32)
- \( \alpha_{f,\text{we}} \) energy content of fat (\( \alpha_{f,\text{we}} = 39.7 \) MJ kg\(^{-1}\), see GfE, 2006, pg. 33)
- \( k_{pf,\text{we}} \) partial efficiency (\( k_{pf,\text{we}} = 0.7 \) MJ MJ\(^{-1}\), see GfE, 2006, pg. 33)
- \( P_{\text{we}} \) constant ratio of protein gain to weight gain (\( P_{\text{we}} = 0.17 \) kg kg\(^{-1}\), see GfE, 2006, pg. 29)
- \( F_{\text{we}, j} \) day-dependent ratio of fat gain to weight gain (in kg kg\(^{-1}\), see below)

According to GfE (2006), pg. 30, the fat content is about 110 g kg\(^{-1}\) for a live weight of 10 kg and 170 g kg\(^{-1}\) for a live weight of 30 kg. From this the following linear equation is deduced:

\[ F_{\text{we}, j} = a_{F,\text{we}} + b_{F,\text{we}} \cdot w_{\text{we}, j} \]  
(11)

where

- \( F_{\text{we}, j} \) day-dependent ratio of fat gain to weight gain (in kg kg\(^{-1}\))
- \( j \) index of day
- \( a_{F,\text{we}} \) constant (\( a_{F,\text{we}} = 0.08 \) kg kg\(^{-1}\))
- \( b_{F,\text{we}} \) constant (\( b_{F,\text{we}} = 0.003 \) kg kg\(^{-1}\) animal)
- \( w_{\text{we}, j} \) mean weaner live weight on day \( j \) (in kg animal\(^{-1}\))

4.2 Cumulative energy requirements and animal weights

According to Equation (7), the energy requirements have to be calculated separately for each single day to obtain the cumulative energy requirements. This would require about 50 calculation steps.

However, Equation (7) can be transformed into an integral equation yielding the cumulative energy requirements in just one calculation step. This transformation presupposes the knowledge about the development of the daily weight gain rate. The latter is a function of animal weight as shown in Figure 1.

![Figure 1: Weaners, weight gain as a function of live weight. Exemplary variable data: squares (LfL, 2004); solid line: linear approximation.](image)

Calculations made for comparison purposes with the total energy summation equation given above show that, without a relevant error, the total energy may also be calculated based on the assumption of a constant (i.e. mean) weight gain rate.

Integration then leads to a relation for the ME amount required for maintenance and growth between animal weights \( w_{\text{we},0} \) and \( w_{\text{we},1} \) (for details see Appendix):

\[ \sum ME_{\text{we}}(w_{\text{we},0}, w_{\text{we},1}) = \eta_{ME,\text{m,we}} \cdot \frac{w_{\text{unit}}^2}{1.75 \cdot g} \]  
(12)

where

- \( \sum ME_{\text{we}}(w_{\text{we},0}, w_{\text{we},1}) \) ME required for the phase between the weights \( w_{\text{we},0} \) and \( w_{\text{we},1} \) (in MJ animal\(^{-1}\) round\(^{-1}\))
- \( \eta_{ME,\text{m,we}} \) specific metabolizable energy required for maintenance (in MJ kg\(^{-1}\) d\(^{-1}\))
- \( w_{\text{unit}} \) unit weight (\( w_{\text{unit}} = 1 \) kg animal\(^{-1}\))
- \( w_{\text{we},0}, w_{\text{we},1} \) animal weights limiting a special phase between weaning and fattening, with \( w_{\text{we},0} < w_{\text{we},1} \) (in kg animal\(^{-1}\))
- \( g \) mean daily weight gain during the entire period between weaning and fattening (in kg animal\(^{-1}\) d\(^{-1}\))
and
\[ \eta_{ME,we}^* = \frac{1}{k_{PF,we}} \cdot \left[ a_{PF,we} \cdot P_{we} + a_{PF,we} \left( a_{F,we} + b_{F,we} \cdot \frac{w_{we,0} + w_{we,1}}{2} \right) \right] \]  

(13)

where

- \( \eta_{ME,we}^* \): effective specific metabolizable energy required for growth, to be used for the entire period between weaning and fattening (in MJ kg\(^{-1}\))
- \( a_{PF,we} \): energy content of protein (\( a_{PF,we} = 23.8\) MJ kg\(^{-1}\), see GfE, 2006, pg. 32)
- \( a_{F,we} \): energy content of fat (\( a_{F,we} = 39.7\) MJ kg\(^{-1}\), see GfE, 2006, pg. 33)
- \( k_{PF,we} \): partial efficiency (\( k_{PF,we} = 0.7\) MJ MJ\(^{-1}\), see GfE, 2006, pg. 33)
- \( P_{we} \): constant ratio of protein gain to weight gain (\( P_{we} = 0.17\) kg kg\(^{-1}\), see GfE, 2006, pg. 29)
- \( a_{F,we} \): constant (\( a_{F,we} = 0.08\) kg kg\(^{-1}\), see Equation (11))
- \( b_{F,we} \): constant (\( b_{F,we} = 0.003\) kg\(^{-1}\) animal, see Equation (11))
- \( w_{we,0}, w_{we,1} \): animal weights limiting a special fattening phase, with \( w_{we,0} < w_{we,1} \) (in kg animal\(^{-1}\))

The results obtained with this one-step calculation procedure are almost identical with the multi-step approach using Equation (7). The difference is in the range of 0.0002 %.

Equation (12) can be used to calculate the energy requirements of single feeding phases. It requires the knowledge of weights rather than days as is the case with Equation (7). This is in line with the fact that information regarding phase feeding of weaners is based on animal weights.

The initial weaner weight is identical to the weight of a suckling-pig at the end of the lactation period. In the inventory, it is assumed to be 8 kg animal\(^{-1}\). The final weight of the weaners’ production phase equals the start weight of the fattening process. It is in the range of about 25 kg animal\(^{-1}\) to 33 kg animal\(^{-1}\). A typical mean value of the daily growth rate is 420 g animal\(^{-1}\) day\(^{-1}\) (KTBL, 2008, pg. 636).

Exemplary total energy requirements for a start weight of 8.5 kg animal\(^{-1}\), an average final weight of 29 kg animal\(^{-1}\) and a mean weight gain of 420 g animal\(^{-1}\) day\(^{-1}\) are 516.4 MJ animal\(^{-1}\).

### 4.3 Comparison with the former approach

Figure 2 displays daily ME requirements data provided in GfE (1987), pg. 54, compared to respective data as calculated with Equation (12) based on GfE (2006).

From Figure 2 we state that for the range of ME requirements typical for the early 1990ies there is no substantial difference between the ME recommendations given in GfE (1987) and GfE (2006). Thus, the inventory makes use of the GfE (2006) calculation method for the entire time series from 1990 onwards.

### 5 Fattening pigs

For fattening pigs, the metabolic energy (ME) requirements comprise the requirements for both maintenance and growth. The total ME required for a fattening period is given by

\[ \Sigma ME_{fp} = \sum_{j=1}^{k_f} \tau_{day,j} \cdot ME_{m,fp,j} + \sum_{j=1}^{k_f} \tau_{day,j} \cdot ME_{g,fp,j} \]  

(14)

where

- \( \Sigma ME_{fp} \): total metabolizable energy required (in MJ animal\(^{-1}\) day\(^{-1}\))
- \( k_f \): number of the final day of the fattening period
- \( \tau_{day,j} \): unit time period (\( \tau_{day} = 1\) day d\(^{-1}\))
- \( j \): index of day
- \( ME_{m,fp,j} \): metabolizable energy required for maintenance for day \( j \) (in MJ animal\(^{-1}\) d\(^{-1}\))
- \( ME_{g,fp,j} \): metabolizable energy required for growth for day \( j \) (in MJ animal\(^{-1}\) d\(^{-1}\))

#### 5.1 Daily energy requirements

The daily energy requirement for maintenance is defined by (cf. discussion Chapter 3.1.2):

\[
\begin{align*}
\Delta ME_{\text{M}} &= \frac{\alpha_f}{\alpha_f + \alpha_m} \cdot ME_{\text{M}} \\
\alpha_f &= \text{constant ratio of protein gain to weight gain} \\
\alpha_m &= \text{constant} \\
ME_{\text{M}} &= \text{constant} \\
\end{align*}
\]
\[ ME_{\text{m,fp},j} = \eta_{\text{ME,m,fp}} \cdot w_{\text{unit}} \left( \frac{w_{b,j}}{w_{\text{unit}}} \right)^{0.75} \]  

where

- \( ME_{\text{m,fp},j} \) is the metabolizable energy required for maintenance for day \( j \) (in MJ animal\(^{-1} \) d\(^{-1} \))
- \( \eta_{\text{ME,m,fp}} \) is the specific metabolizable energy required for maintenance (in MJ kg\(^{-1} \) d\(^{-1} \), see below)
- \( w_{\text{fp},j} \) is the live weight on day \( j \) (in kg animal\(^{-1} \))
- \( w_{\text{unit}} \) is the unit weight \( (w_{\text{unit}} = 1 \text{ kg animal}^{-1}) \)

According to GfE (2006), pg. 23, the specific energy for maintenance can be described by

\[ \eta_{\text{ME,m,fp}} = \eta_{\text{ME,m,fp},a} \cdot \left( a_m - b_m \cdot \frac{w_{b,j} - w_b}{w_a - w_b} \right) \]  

where

- \( \eta_{\text{ME,m,fp}} \) is the specific metabolizable energy required for maintenance (in MJ kg\(^{-1} \) d\(^{-1} \))
- \( \eta_{\text{ME,m,fp},a} \) is the reference value of specific metabolizable energy for maintenance \( (\eta_{\text{ME,m,fp},a} = 0.44 \text{ MJ kg}^{-1} \text{ d}^{-1}) \)
- GfE, 2006, pg. 23)
- \( a_m \) is the constant \( (a_m = 1.25 \text{ MJ MJ}^{-1}) \)
- \( b_m \) is the constant \( (b_m = 0.25 \text{ MJ MJ}^{-1}) \)
- \( w_{b,j} \) is the live weight on day \( j \) (in kg animal\(^{-1} \))
- \( w_a, w_b \) are the thresholds animal weights (in kg animal\(^{-1} \), see below)

GfE (2006) uses \( w_a = 30 \text{ kg animal}^{-1} \) and \( w_b = 100 \text{ kg animal}^{-1} \). GfE (2006) applies Equation (16) also to animal weights falling below \( w_a \) and exceeding \( w_b \). Then, in case of animal weights smaller than \( w_b \), the term in brackets is set equal to \( a_m \) and for weights larger than \( w_b \), it equals \( a_m - b_m \).

The energy required for growth is calculated according to:

\[ ME_{\text{g,fp},j} = \eta_{\text{ME,g,fp},j} \cdot \left( \frac{\Delta W}{\Delta t} \right)_{fp,j} \]  

where

- \( ME_{\text{g,fp},j} \) is the metabolizable energy required for daily growth on day \( j \) (in MJ animal\(^{-1} \) d\(^{-1} \))
- \( j \) is the index of day
- \( \eta_{\text{ME,g,fp},j} \) is the specific metabolizable energy required for growth on day \( j \) (in MJ kg\(^{-1} \), see below)
- \( \Delta W/\Delta t \) is the daily weight gain per animal and day \( j \) (in kg animal\(^{-1} \) d\(^{-1} \))

The specific energy requirements for growth, \( \eta_{\text{ME,g,fp},j} \), is a function of protein and fat gains, which means, it is also a function of life time:

\[ \eta_{\text{ME,g,fp},j} = \frac{\alpha_p}{k_p} \cdot P_j + \frac{\alpha_f}{k_f} \cdot F_j \]  

where

- \( \eta_{\text{ME,g,fp},j} \) is the specific metabolizable energy required for growth on day \( j \) (in MJ kg\(^{-1} \))
- \( j \) is the index of day
- \( \alpha_p \) is the energy content of protein (\( \alpha_p = 23.8 \text{ MJ kg}^{-1} \))
- \( \alpha_f \) is the energy content of fat (\( \alpha_f = 39.7 \text{ MJ kg}^{-1} \))
- \( k_p \) is the partial efficiency (\( k_p = 0.56 \text{ MJ MJ}^{-1} \))
- \( k_f \) is the partial efficiency (\( k_f = 0.74 \text{ MJ MJ}^{-1} \))
- \( P_j \) is the day-dependent ratio of protein gain to weight gain (in kg kg\(^{-1} \), see below)
- \( F_j \) is the day-dependent ratio of fat gain to weight gain (in kg kg\(^{-1} \), see below)

The relative protein gain \( P \) is obtained from the Equation (8) in GfE (2006) differentiated with respect to animal weight.

\[ P_j = a_p - b_p \cdot w_{fp,j} \]  

where

- \( P_j \) is the day-dependent ratio of protein gain to weight gain (in kg kg\(^{-1} \), see below)
- \( j \) is the index of day
- \( a_p \) is the constant \( (a_p = 0.168 \text{ kg kg}^{-1}) \)
- \( b_p \) is the constant \( (b_p = 0.0001828 \text{ kg kg}^{-1} \text{ animal}) \)
- \( w_{fp,j} \) is the live weight on day \( j \) (in kg animal\(^{-1} \))

Differention of Equation (9) in GfE (2006) with respect to animal weight yields the relative fat gain \( F \). A correction factor of 0.94 kg kg\(^{-1} \) has to be taken into account (GfE, 2006, pp. 28 and 31).

\[ F_j = a_f \cdot (b_f + c_f \cdot w_{fp,j}) \]  

where

- \( F_j \) is the day-dependent ratio of fat gain to weight gain (in kg kg\(^{-1} \))
- \( j \) is the index of day
- \( a_f \) is the constant \( (a_f = 0.94 \text{ kg kg}^{-1}) \)
- \( b_f \) is the constant \( (b_f = 0.1162 \text{ kg kg}^{-1}) \)
- \( c_f \) is the constant \( (c_f = 0.002778 \text{ kg kg}^{-1} \text{ animal}) \)
- \( w_{fp,j} \) is the live weight on day \( j \) (in kg animal\(^{-1} \))

5.2 Cumulative energy requirements and animal weights

By analogy to the procedure used for weaners (see Chapter 4.2) the summation formula (14) is transformed into an equation that allows to calculate the cumulative metabolizable energy by one single calculation step rather
than summing up daily energy requirements with more than a hundred single steps (for details of the transformation process see Appendix).

In order to account for the fact that the specific metabolizable energy $\eta_{\text{ME},\text{m,n}}$ is defined separately for the animal weight intervals “falling below 30 kg animal$^{-1}$”, “30 to 100 kg animal$^{-1}$” and “exceeding 100 kg animal$^{-1}$”, the transformation has to be performed separately for these intervals.

As for weaners, the transformation is performed under the assumption of a constant (i.e. mean) weight gain rate (cf. also Dämgen et al., 2009, pg. 178). Integration then leads to an equation for the amount of ME required for the growth between animal weights $w_0$ and $w_1$, where $w_0$ and $w_1$ represent $w_{q_0}$ and $w_{q_1}$, respectively:

$$\Sigma ME_0(w_0, w_1) = \eta_{\text{ME},\text{m,n}} \cdot \left[ \varphi_1(w_0, w_1, w_\lambda) + \varphi_2(w_0, w_1, w_A, w_B) + \varphi_3(w_0, w_1, w_B) \right] + \eta_{\text{ME},\text{g}} \cdot (w_1 - w_0)$$  \hspace{1cm} (21)

where

- $ME_0(w_0, w_1)$ ME required for the fattening phase between the weights $w_0$ and $w_1$ (in MJ animal$^{-1}$ ME)
- $\eta_{\text{ME},\text{m,n}}$ reference value of specific metabolizable energy for maintenance ($\eta_{\text{ME},\text{m,n}} = 0.44$ MJ kg$^{-1}$ d$^{-1}$, GfE, 2006, pg. 23)
- $w_0, w_1$ animal weights limiting a special fattening phase, with $w_0 < w_1$ (in kg animal$^{-1}$ ME)
- $w_A, w_B$ thresholds of animal weight (in kg animal$^{-1}$, see Chapter 5.1)
- $g$ mean daily weight gain during the entire fattening period (in kg animal$^{-1}$ d$^{-1}$)

and

$$\varphi_1 = a_2 \cdot \left( \frac{\min(w_1, w_A)}{w_\text{unit}} \right)^{1.75} \left[ \frac{\min(w_0, w_A)}{w_\text{unit}} \right]^{1.75}$$  \hspace{1cm} (22)

$$\varphi_2 = \frac{w_\text{unit}^2}{1.75} \left( a_m \cdot b_m \cdot w_A \right) \left[ \frac{\max(w_A, \min(w_1, w_B))}{w_\text{unit}} \right]^{1.75} \left[ \frac{\min(w_B, w_A)}{w_\text{unit}} \right]^{1.75} - \frac{b_m \cdot w_A^3}{2.75 \cdot (w_B - w_A)} \left[ \frac{\max(w_A, \min(w_1, w_B))}{w_\text{unit}} \right]^{2.75} \left[ \frac{\min(w_B, w_A)}{w_\text{unit}} \right]^{2.75}$$  \hspace{1cm} (23)

$$\varphi_3 = \frac{w_\text{unit}^2}{1.75} \left( \frac{\max(w_1, w_B)}{w_\text{unit}} \right)^{1.75} - \left( \frac{\max(w_0, w_B)}{w_\text{unit}} \right)^{1.75}$$  \hspace{1cm} (24)

$$\eta_{\text{ME},\text{g}} = \frac{\alpha_p}{\kappa_p} \left( a_p \cdot b_p \cdot \frac{w_0 + w_1}{2} + a_{\text{f}} \cdot a_F \cdot \left( b_F + c_F \cdot \frac{w_0 + w_1}{2} \right) \right)$$  \hspace{1cm} (25)

where

- $\varphi_1, \varphi_2, \varphi_3$ functions of $w_p, w_1, w_A, w_B$ (in kg$^2$ animal$^{-2}$)
- $w_0, w_1$ animal weights limiting a special fattening phase, with $w_0 < w_1$ (in kg animal$^{-1}$)
- $w_A, w_B$ animal weight thresholds (in kg animal$^{-1}$, see Chapter 5.1)
- $w_\text{unit}$ unit weight ($w_\text{unit} = 1$ kg animal$^{-1}$)
- $a_m, b_m$ constant ($a_m = 1.25$ MJ MJ$^{-1}$, GfE, 2006, pg. 23)
- $b_F$ constant ($b_F = 0.25$ MJ MJ$^{-1}$, GfE, 2006, pg. 23)
- $\alpha_p, \kappa_p$ mean specific metabolizable energy required for growth, see above (in MJ kg$^{-1}$)
- $\alpha_p$ coefficient ($\alpha_p = 23.8$ MJ kg$^{-1}$, see GfE, 2006, pg. 32)
- $\kappa_p$ coefficient ($\kappa_p = 0.56$ MJ MJ$^{-1}$, see GfE, 2006, pg. 33)
- $a_F$ constant ($a_F = 0.168$ kg kg$^{-1}$, see above)
\[ h_p \text{ constant (}h_p = 0.0001828 \text{ kg}^{-1} \text{ animal, see above)} \]
\[ a_f \text{ coefficient (}a_f = 39.7 \text{ MJ kg}^{-1} \text{, see GfE, 2006, pg. 33)} \]
\[ k_f \text{ coefficient (}k_f = 0.74 \text{ MJ MJ}^{-1} \text{, see GfE, 2006, pg. 33)} \]
\[ a_r \text{ constant (}a_r = 0.94 \text{ kg kg}^{-1} \text{, see above)} \]
\[ b_f \text{ constant (}b_f = 0.1162 \text{ kg kg}^{-1} \text{, see above)} \]
\[ c_f \text{ constant (}c_f = 0.002778 \text{ kg}^{-1} \text{ animal, see above)} \]

Again, exemplary comparisons between the results obtained with the application of Equations (14) and (21) ff. shows a perfect agreement (relative deviation 0.0001 %).

According to the data compilation in Haenel et al. (2010), Chapter 5.5.1.2, the initial weight of fattening pigs is in the range of about 25 kg animal\(^{-1}\) to 33 kg animal\(^{-1}\) while the final weight is between 105 kg animal\(^{-1}\) and 120 kg animal\(^{-1}\); averaging the daily growth rate over the two decades from 1990 on and over all federal states yields about 680 g animal\(^{-1}\) d\(^{-1}\).

Exemplary total energy requirements for a mean start weight of 29 kg animal\(^{-1}\), a mean final weight of 113 kg animal\(^{-1}\) and a mean daily weight gain of 680 g animal\(^{-1}\) d\(^{-1}\) (with a resulting fattening period duration of 124 days) are 3,3 GJ animal\(^{-1}\).

5.3 Comparison with the former approach

Figure 3 displays daily ME requirements data provided in GfE (1987), pg. 35, compared to respective data as calculated with Equation (21) according to GfE (2006).

The ME requirements as calculated using the GfE (1987) approach exceed those obtained with Equation (21) based on GfE (2006) by up to 12 %. GfE (2006), pg. 27 ff, explains this with the different experimental data available. However, the 2006 GfE edition is based on data published since 1980 which leads to the assumption that the energy requirements recommendations in this edition should be appropriate to describe the situation in the years covered by the inventory, i.e. the past two decades.

6 Breeding boars

For breeding boars, the metabolic energy requirements comprise the requirements for both maintenance and growth. However, as the contribution of boars to the overall emissions from pig husbandry is almost negligible, the inventory simply assumes all boars counted by the official survey to be breeding boars whose weight gain is restricted (GfE, 2006, pg. 84).

Due to the breeding boars’ limited importance and the scarcity of data (GfE, 2006, pg. 84) the inventory does not attempt to model energy requirements of boars. While GfE (1987), pg. 68, suggested mean ME requirements of 30 MJ animal\(^{-1}\) d\(^{-1}\), GfE (2006), pg. 84, recommends a range of 30 to 35 MJ animal\(^{-1}\) d\(^{-1}\). A rough estimate based on the data provided in GfE (2006), pg. 83, and the use of ME calculation approach for fattening pigs indicates that the value of 35 MJ animal\(^{-1}\) d\(^{-1}\) matches a mean boar weight of about 200 kg animal\(^{-1}\) (expert judgment E. Schulz) better than 30 MJ animal\(^{-1}\) d\(^{-1}\). Hence the inventory assumes daily ME requirements of 35 MJ animal\(^{-1}\) d\(^{-1}\) leading to cumulative annual requirements of 12775 MJ animal\(^{-1}\) a\(^{-1}\).

7 Conclusions

It is good practice to use the best data available to construct emission inventories. For pigs, the national situation had formerly been reflected in a data set published in GfE (1987) with background information dating from the 1970s and early 1980s. The updated methodologies and data published in GfE (2006) were a remarkable step forward with respect to the reliability and accuracy of the German agricultural emission inventory. The use of equations rather than tables allows for a simpler treatment of variable energy requirement calculations.

For the most important subcategory, i.e. fattening pigs, the application of the updated GfE (2006) methodology results in reduced energy requirements, hence in reduced excretion rates for faeces and urine and subsequently reduced emissions. The opposite is the case for sows, wean-
ers and boars, but this is outweighed by the reductions achieved for fattening pigs.

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Appendix: Derivation of a closed mathematical description of the cumulative ME requirements of weaners

Cumulative energy requirements can be calculated by adding up daily requirements over the time span considered:

\[
\Sigma ME_{we} = \sum_{j=1}^{k_\text{fin}} \tau_{\text{day},j} \cdot ME_{we,j}
\]  
(A1)

where

- \(\Sigma ME_{we}\): total metabolizable energy required between weaning and fattening (in MJ animal\(^{-1}\) round\(^{1}\) ME)
- \(\tau_{\text{day}}\): unit time period (\(\tau_{\text{day}} = 1 \text{ d round}\(^{1}\) )
- \(j\): index of day
- \(k_\text{fin}\): number of the final day of the weaners’ life span
- \(ME_{we,j}\): metabolizable energy required for maintenance and growth on day \(j\) (in MJ animal\(^{-1}\) d\(^{-1}\) ME)

In the German agricultural inventory, equations had been derived using data in GfE (1987) that expressed a sum like Equation (A1) in a single equation using a fitted power series (Dämmgen et al., 2007). The latter had to be replaced with the use of new data according to GfE (2006). It was decided to base the derivation of a new single equation on mathematical integration instead of fitting power series.

As a preparation for the mathematical solution, Equation (A1) has to be rewritten as difference quotient:

\[
\Delta (\Sigma ME_{we}) \over \Delta \tau\bigg|_j = ME_{we,j} = ME(w_{we,j})
\]  
(A2)

where

- \(\Delta (\Sigma ME_{we})\): increment of total metabolizable energy required between weaning and fattening (in MJ animal\(^{-1}\) round\(^{1}\) ME)
- \(\Delta \tau\): time increment (\(\Delta \tau = \tau_{\text{day}} = 1 \text{ d round}\(^{1}\) )
- \(j\): index of day
- \(ME_{we,j}\): metabolizable energy required for maintenance and growth on day \(j\) (in MJ animal\(^{-1}\) d\(^{-1}\) ME)
- \(w_{we,j}\): mean weaner live weight on day \(j\) (in kg animal\(^{-1}\) )

It is plausible that this procedure is valid not only for daily time steps but also for infinitesimally small time steps. Hence the difference quotient can be transformed in a differential quotient for each time \(\tau_i'\). Here, \(\tau_i'\) is a compartmentalized time scale (in s round\(^{1}\)) that is connected to the timescale \(\tau\) (in d round\(^{1}\)) by a units conversion factor.

\[
\frac{d(\Sigma ME_{we})}{d\tau'}\bigg|_{\tau_i'} = \gamma^{-1} \cdot ME(w_{we}(\tau_i')) = f(\tau_i')
\]  
(A3)

where

- \(\Sigma ME_{we}\): increment of total metabolizable energy required between weaning and fattening (in MJ animal\(^{-1}\) round\(^{1}\) ME)
- \(\tau\): time (in s round\(^{1}\))
- \(\tau_i'\): specific time (in s round\(^{1}\))
- \(\gamma\): time units conversion factor (\(\gamma = 86400 \text{ s d}^{-1}\))
- \(w_{we}(\tau_i')\): mean weaner live weight at time \(\tau_i'\) (in kg animal\(^{-1}\))

and

\[
f(\tau_i') = \gamma^{-1} \cdot \{a \cdot (w_{we})^b + [c + (d \cdot w_{we} + e)] \cdot g\}
\]  
(A4)

where

- \(f(\tau_i')\): metabolizable energy required per animal and second (in MJ animal\(^{-1}\) s\(^{-1}\) ME)
- \(\gamma\): time units conversion factor (\(\gamma = 86400 \text{ s d}^{-1}\))
- \(a, b, c, d, e, g\): see text below
- \(w_{we}(\tau_i')\): time dependent mean weaner live weight (in kg animal\(^{-1}\))

The entities \(a, b, c, d, e\) and \(g\) originate from the comparison of the equations describing the energy requirements for maintenance and growth (Chapter 4.1). In particular, \(g\) denotes a constant growth rate.

Integration of \(f(\tau_i')\) leads to the cumulative energy requirements:

\[
\Sigma ME_{we} = \int_{0}^{\tau_i'} f(w_{we}(\tau_i')) \, d\tau'
\]  
(A5)

where

- \(\Sigma ME_{we}\): total metabolizable energy required between weaning and fattening (in MJ animal\(^{-1}\) round\(^{1}\) ME)
- \(\tau\): time (in s round\(^{1}\))
- \(f(w_{we}(\tau_i'))\): metabolizable energy required per animal and second (in MJ animal\(^{-1}\) s\(^{-1}\) ME)
- \(w_{we}(\tau_i')\): time dependent mean weaner live weight (in kg animal\(^{-1}\))

and

\[
\tau_i' = \gamma \cdot k_\text{fin} \cdot \tau_{\text{day}}
\]  
(A6)

where

- \(\tau_i'\): time of the end of the weaners’ life span (in s round\(^{1}\))
time units conversion factor ($\gamma = 86400 \text{ s}^{-1}$)

number of the final day of the weaners’ lifespan

unit time period ($t_{day} = 1 \text{ d round}^{-1}$)

The entity $g$ can be used in Equation (A5) to substitute the independent variable time by weight

$$\sum ME_{\text{we}} = \frac{\gamma}{g} \int \frac{f(w_{\text{we}})}{w_{\text{we}, 0}} \, dw_{\text{we}} \quad (A7)$$

where

$\sum ME_{\text{we}}$ total metabolizable energy required between weaning and fattening-period (in MJ animal$^{-1}$ round$^{-1}$ ME)

$\gamma$ time units conversion factor ($\gamma = 86400 \text{ s}^{-1}$)

$g$ animal weight gain per unit time (in kg animal$^{-1}$ day$^{-1}$)

$f(w_{\text{we}})$ metabolizable energy required per animal and second (in MJ animal$^{-1}$ s$^{-1}$ ME)

$w_{\text{we}}$ mean weaner live weight (in kg animal$^{-1}$)

$w_{\text{we}, 0}$ mean weaner live weight at weaning (in kg animal$^{-1}$)

$w_{\text{we}, 1}$ mean weaner live begin at the end of the fattening period (in kg animal$^{-1}$)

Execution of the integral and subsequent re-substitution of the entities $a, b, c, d, e$ and $g$ in $f(w_{\text{we}})$ results in Equation (12) in Chapter 4.2.

References


