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Determining the Constitutive Parameters of a Macro-scale Second-Gradient Model for Planar Pantographic Structures by Using Optimization Algorithms

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Abstract. Pantographic structures have been proposed as a group of metamaterials that show high toughness in extension. Modeling such structures is technically possible through microscopic first-order continuum theories by using a suitably small length-scale, although the computational cost would be high. The aim of this research is to estimate the constitutive parameters of a planar pantographic structure by means of an optimization process. A previously-proposed macroscopic second-gradient model which is characterized by deformation energy is used for modeling the mechanical behavior of the structure. The macroscopic model will be developed based on the results of the numerical simulations of a microscopic model. In this problem, an evolutionary multi-objective optimization algorithm is utilized which minimizes the squared error of the outputs in order to determine the parameters of the panthographic structure.

Introduction

Metamaterials are referred to a group of materials that are designed carefully to have tailored properties. Pantographic structures are an example of metamaterials that are proposed as a group of structures to have high toughness and high strength-to-weight ratio which makes them attractive for industrial and aerospace applications [4]. Figure 1 shows an example of a pantographic structure. The structure is made up of straight beams which are connected together, at their intersection points, by cylinder pivots. These pivots play a significant role in absorbing large amounts of energy by restricting the relative motion of fibers. It is possible to model pantographic structures using classical continuum mechanics, which is based on Cauchy theories, and describe the mechanical behavior in detail. However, it will not be efficient in terms of computational cost. An appropriate solution is to employ "Generalized Continuum Mechanics" in order to capture the effects of the substructure at a scale which is large enough to neglect the geometrical complexities of the micro-structure.
The goal of the present work is to identify the constitutive parameters of a macro-scale model for a planar pantographic structure in a way that the inspected properties of the structure matches well with that of the data obtained, using Cauchy model, from an exact 3D model. This model will be a tool for calculation and prediction of the mechanical behavior of the structure. As a continuation to perivous studies on numerical identifications [1, 3, 4], here, an optimization algorithm is exploited for an automated and easier parameter identification process.
Different forms of Genetic Algorithm (GA), which is among evolutionary algorithms, has been developed for dealing with optimization problems, such as Non-Dominated Sorting Genetic Algorithm (NSGA) [7] and NSGA-II [2]. The NSGA-II yields surprisingly better results compared the other two since it uses two criteria for ranking solutions: grouping the objective function values in 'Fronts' and calculating the crowding distance for values of each 'Front' [2].
In this research, we carry out the constitutive parameter identification of a planar pantographic structure in two cases. First, the optimization is done with one 'fitness' function using GA and then with two objective functions using NSGA-II.

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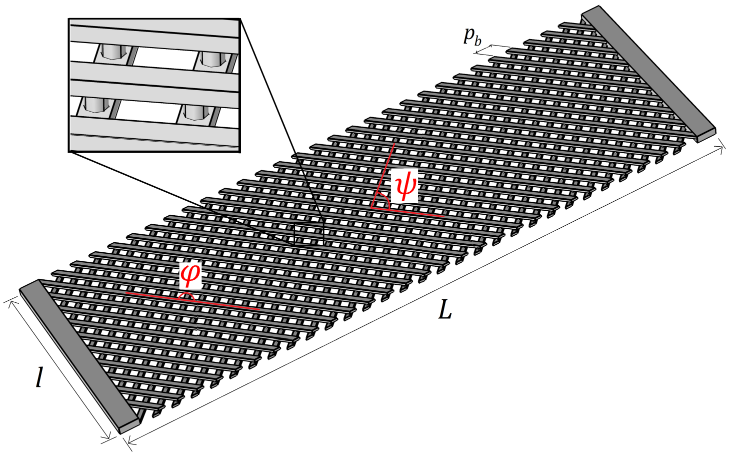

Figure 1. A pantographic structure and its geometrical parameters

1 Macro Model and Geometry

For describing the pantographic structure at macro level, the second-gradient model presented in [3] is considered here. In this model, the current position x of a material point is a function of the position $\mathbf{X}$ in the reference configuration through the map $\chi : \ \Omega \rightarrow \mathbf{R}^2$ where Ω is a rectangular plane region.

$$x = \chi(\mathbf{X}) = (X^\alpha + u^\alpha(X^\beta))\mathbf{D}_\alpha \tag{1}$$

The deformation gradient is $F = \nabla\chi$ and the field of unit vectors tangent to the two families of fibers is $e_\alpha = \mathbf{F}\mathbf{D}_\alpha/||\mathbf{F}\mathbf{D}_\alpha||$ where $\mathbf{D}_\alpha$ is a Lagrangean basis, $\alpha = \{1, 2\}$. The auxiliary vector field c_α is:

$$c_\alpha = \frac{\nabla\mathbf{F}|\mathbf{D}_\alpha \otimes \mathbf{D}_\alpha}{||\mathbf{F}\mathbf{D}_\alpha||} \qquad \text{where } (\nabla\mathbf{F}|\mathbf{D}_\alpha \otimes \mathbf{D}_\alpha)^\beta = \partial_\alpha F^\beta_\alpha = \partial_{\alpha\alpha}\chi^\beta \tag{2}$$

The strain measures, stretch of fibers ($\varepsilon_\alpha = \mathbf{F}|\mathbf{D}_\alpha - 1$), fiber curvature ($\kappa_\alpha = ||c_\alpha - (c_\alpha.e_\alpha)e_\alpha|| = ||(\mathbf{I} - e_\alpha \otimes e_\alpha)c_\alpha||$), and shear distortion γ ($\sin(\gamma) = e_1.e_2$) are introduced. The strain energy density that considers all the above contributions is assumed to be as follows.

$$W_M(\varepsilon_\alpha, \kappa_\alpha, \gamma) = \sum_\alpha (\frac{1}{2}K_I\varepsilon_\alpha^2 + \frac{1}{2}K_{II}\kappa_\alpha^2) + \frac{1}{2}K_p\gamma^2, \qquad K_I, K_{II}, K_p > 0, \ \text{constant} \tag{3}$$

The governing equations are obtained using a variational principle:

$$\delta \int_\Omega W_M(\varepsilon_\alpha, \kappa_\alpha, \gamma)d\Omega = 0 \quad \forall \ \delta u \tag{4}$$

The pantographic structure shown in Fig. 1 is considered for modeling in this study. The geometrical parameters, shown in Fig.1, are as follows: $L = 0.2096$ m, $l = 0.06987$ m, $p_b = 4.85$ mm and the height of each beam, $h_b = 0.9$ mm, the width of each beam, $b_b = 1.6$ mm, the height of each pivot, $h_p = 1$ mm, and the diameter of each pivot $d_p = 1.2$ mm. The material used for the structure is Polyamide PA 2200 (Young Modulus = 1600 Mpa, Poisson's ratio = 0.3, and mass density = 930 kg/m^3). Bias extension tests are simulated on the structure in COMSOL Multiphysics FEM enviroment and the total stored energy and the angles ψ and φ are calculated. ψ is the angle between two beams intersecting at the center of the specimen and φ is the bending angle of the beam that meets the corner of the specimen, see Fig. 1. The reason of choosing these two angles is that ψ and φ are the representatives of the shear energy in the central region and the bending energy in the most bent region respectively [3].

2 Optimization of the Macro Model Parameters

In this section, the considered results of the macro model are fitted with the micro model results through an optimization process. The constitutive parameters of the macro model, K_I, K_{II}, and K_p, are determined by the optimization algorithm. At first, we consider the squared normalized error of the outputs

(The total stored energy and the two angles) as the 'fitness' function (f_1) and minimize it using GA. Then, we defined another objective function, which grants double significance to ψ compared to φ (Eq. (5)). This is due to the fact that the energy involved in ψ angle is mainly governed by K_p [3] and on the other hand by inspecting the energy components, we can see that the shear energy has the largest portion of the total stored energy. Therefore, if the predicted ψ has lower error, the chance of predicting more accurate energy values is higher. NSGA-II is utilized for the new optimization problem which has two objectives. For investigating the effect of defining the new objective function, NSGA-II is modeled with the same optimization algorithm parameters.

$$f_2 = 2\sum_{i}^{n}(\frac{\psi_i^M - \psi_i^m}{\psi_i^m})^2 + \sum_{i}^{n}(\frac{\varphi_i^M - \varphi_i^m}{\varphi_i^m})^2 \tag{5}$$

where i is the index of the i-th step loading and M and m denote the quantities from macro and micro models, respectively.

In NSGA-II, the group of objective function value(s) that dominate other values are preferred, and they are called the Pareto Front. The fronts are chosen using a matrix. The objective function values are plotted on a graph with the axes showing the first and second objective function values. The advantage of Pareto Front is that a set of optimum points can be assessed. The importance of having a set of optimum solutions is that one can choose the optimum point even in case of occurring unexpected limitations [2].

Figure 2(a) shows the Pareto Front and the Fig.2 (b) shows the number of points in Pareto Front in the ten first iterations. In complicated cases of multi-objective optimization problems, in case of applying NSGA-II, the number of points in Pareto Front fall to zero after a few iterations and then suddenly it increases as much as the number of all point. In this condition, NSGA-II acts poorly because of loss of selection pressure in objective function evaluation. For overcoming this problem and increasing the accuracy of ranking, the 'dominance' concept was replaced by 'Strictly dominance', 'Weakly dominance', and 'Indifferent' concepts through Fuzzy methods. Later, the Fuzzy-based Pareto Optimality was presented using the Left Gaussian function and the Fuzzy-Pareto-Dominance [5, 6]. Since the plot of Pareto Front is desired and the algorithm performs well in high iterations, our optimization has not encountered the above-mentioned problem. The results show that the population is not high enough to have a Pareto Front with less points than the population. On the other hand, considering the computational cost of the model, it is not efficient to increase the population.

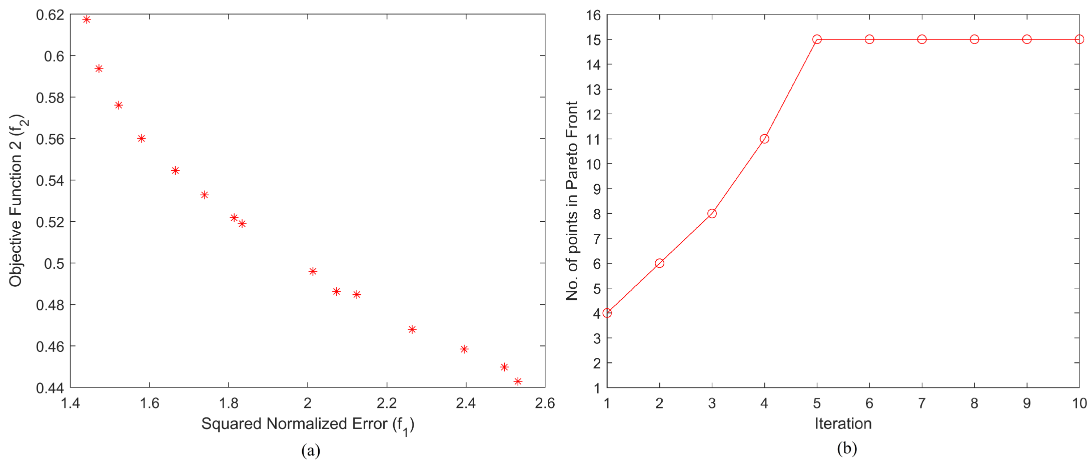

Figure 2. (a)The Pareto Front and (b)number of points in Pareto Front in every iteration

3 Results

By means of the optimization process, the constitutive parameters of the planar macro model are identified. In the first optimization, with one fitness function, the average value of the squared normalized error of the optimum point was 1.455 and then it was slightly decreased by defining another objective function.The constitutive parameters calculated by the GA are $K_I = 6.274 \times 10^5$ (N/m), $K_{II} = 7.81 \times 10^-2$ (Nm), and $K_p = 3.684 \times 10^3$ (N/m). In NSGA-II, since it is a multi-objective optimization, a single set of parameters is not introduced as the optimum point, in other words, we have a number of optimum points in the Pareto that non of them dominate the others, therefore, we may choose any of the points with regard to the problem conditions. The values of the stored energy and the angles ψ and φ calculated by simulating a bias extension test of the pantographic structure using the planar second-gradient macro model are compared (in Fig. 3) with the values from Cauchy continuum micro model which were previously calculated in [4] for the same structure. All the calculated values show acceptable accordance except for the stored energy after 0.04m of displacement, where we see a discrepancy.

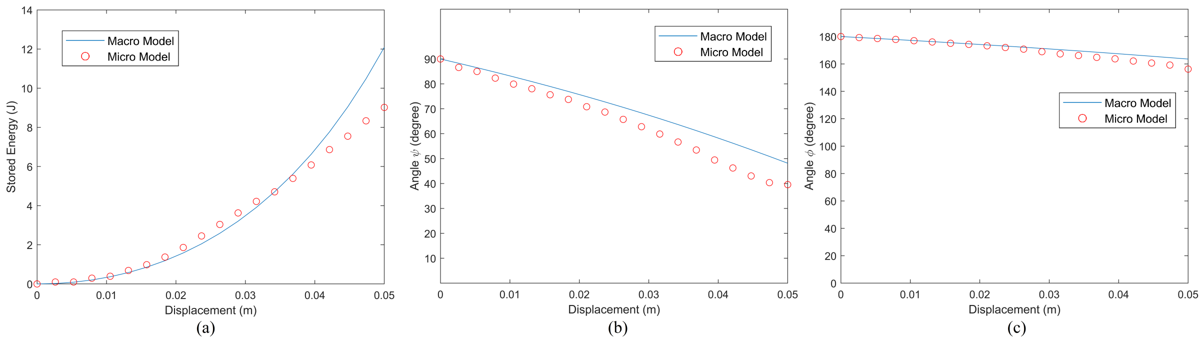

Figure 3. Comparison of the values of (a)Stored Energy, (b)The angle ψ, and (c)The angle φ from the micro[4] and macro models

4 Conclusion

In this paper, a multi-objective optimization algorithm is utilized for identifying the constitutive parameters of a second-gradient macro model for a planar pantographic structure by minimizing the error of the outputs of the model. By the virtue of the automated algorithm, the reduced-order model could be created for different structures more easily.

References

[1] Michele De Angelo, Emilio Barchiesi, Ivan Giorgio, and B. Emek Abali. Numerical identification of constitutive parameters in reduced-order bi-dimensional models for pantographic structures: application to out-of-plane buckling. *Archive of Applied Mechanics* (2019), pp. 1–26.

[2] Kalyanmoy Deb, Samir Agrawal, Amrit Pratap, and Tanaka Meyarivan. A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II. In: *International conference on parallel problem solving from nature*. Springer. 2000, pp. 849–858.

[3] Francesco dell'Isola, Ivan Giorgio, Marek Pawlikowski, and Nicola Luigi Rizzi. Large deformations of planar extensible beams and pantographic lattices: heuristic homogenization, experimental and numerical examples of equilibrium. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 472.2185 (2016), p. 20150790.

[4] Ivan Giorgio. Numerical identification procedure between a micro-Cauchy model and a macro-second gradient model for planar pantographic structures. *Zeitschrift für angewandte Mathematik und Physik* 67.4 (2016), p. 95.

[5] Zhenan He, Gary G Yen, and Jun Zhang. Fuzzy-based Pareto optimality for many-objective evolutionary algorithms. *IEEE Transactions on Evolutionary Computation* 18.2 (2013), pp. 269–285.

[6] Mario Köppen, Raul Vicente-Garcia, and Bertram Nickolay. Fuzzy-pareto-dominance and its application in evolutionary multi-objective optimization. In: *International Conference on Evolutionary Multi-Criterion Optimization*. Springer. 2005, pp. 399–412.

[7] Nidamarthi Srinivas and Kalyanmoy Deb. Muiltiobjective optimization using nondominated sorting in genetic algorithms. *Evolutionary computation* 2.3 (1994), pp. 221–248.