

A gravity equation for commuting with an application to estimating regional border effects in Belgium

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Abstract

This article derives a gravity equation for commuting and uses it to identify the effect of regional borders on commuting. We build on the seminal trade paper by Anderson and Van Wincoop (2003, Gravity with gravitas: a solution to the border puzzle. *The American Economic Review*, 93: 170–192) and highlight some interesting similarities between our model and Wilson's doubly constrained gravity equation [Wilson, A. (2010) Entropy in urban and regional modelling: retrospect and prospect. *Geographical analysis*, 42: 364–394], a workhorse model from spatial interaction theory. The model is estimated by applying a negative binomial regression method on Belgian inter-municipal commuting data. We show that regional borders exert a sizeable residual deterrent effect on commuting, a finding with obvious implications for regional labour market integration. This border effect differs significantly between regions and depends on the direction in which the border is crossed.

Keywords: Gravity equation, border effect, commuting, spatial interaction theory

JEL classifications: J61, F10, R12

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1. Introduction

Commuting is an important spatial equilibrating mechanism in the labour market. In standard closed-economy labour market models, commuting reduces disparities in regional labour market outcomes such as unemployment rates and wages, and brings aggregate welfare gains (see, for example, Borjas, 2001). Commuting is costly, however. One can think of obvious costs that are directly related to commuting distance, such as straightforward travel expenses or the opportunity cost of a lengthy daily commute. Additionally, there exist less tangible but nonetheless substantial costs when a worker commutes to a different region. These costs could arise from, for example, informational deficiencies, linguistic barriers or a regional cultural divide. They explain the difference between the expected commuting flows between regions based on purely economic and geographic factors and observed commuting flows. Such 'missing interregional commuting' suggests an inefficient spatial allocation of labour, implying that welfare gains can be obtained from policies aimed at removing these barriers, such as improving information exchange related to interregional job search, adjusting the regional skill structure, investing in language education, etc. This should be specifically

beneficial for countries with marked differences in regional labour market performance, as is the case for many European countries.

This article quantifies the effect of regional borders on commuting by means of a gravity framework and while doing so bridges the gap between the gravity traditions developed in the context of international trade on the one hand and spatial interaction modelling on the other. Our gravity equation is derived from a small spatial labour market model inspired by Anderson and Van Wincoop (2003) (AvW), in which firms characterized by a love-of-variety production function employ workers from different locations. In the spatial interaction literature, gravity equations have mostly been set up without micro-foundations and have rather relied on analogies from statistical mechanics (see Wilson, 2010, for a review) or on discrete choice theory (Anas, 1983; Fotheringham, 1986). The development of gravity equations in the context of international trade has taken a different route, relying on economic models where demand and/or supply are derived from optimizing behaviour of economic agents and the gravity equation describes the resulting market-clearing flow of goods. Applying the latter approach in a labour market setup results in a functional form for the gravity equation which is remarkably similar to the functional form of the doubly constrained specification often used in spatial interaction modelling, but nevertheless differs from it in some important aspects. Measures of fit suggest that our approach improves commuting flow predictions. We also argue that it enables us to more accurately identify the border effect by taking into account economic push and pull factors.

The commuting gravity equation is empirically estimated by means of a count model. Count models allow for zero as a possible outcome and avoid the biases introduced by estimating log-linearized models in the presence of heteroskedasticity (Silva and Tenreyro, 2006). The empirical application uses aggregate data on commuting flows between 580 Belgian municipalities¹ in 2008. Belgium is an interesting case for the study at hand for a number of reasons:

Regional borders are important in Belgium. Belgium is a multilingual country, consisting of three NUTS1 regions; Flanders in the north and Wallonia in the south are officially unilingually Dutch and French speaking regions, respectively. The central capital region of Brussels is officially bilingual, but de facto a majority of the local population speaks French (Janssens, 2008). Nevertheless, many jobs in Brussels require knowledge of both French and Dutch. Belgium is a federal state, with regional governments in each of the three NUTS1 regions. Successive reforms of the Belgian state resulted in an increasing degree of independence for the regions, for example, with regards to active labour market policy. The socio-cultural divide between the regions is large: with the exception of the capital region of Brussels, there exist no cross-regional political parties which are represented in the national parliament and none of the dominant media channels in the country target all three regions.

1 Nine municipalities belonging to the small German speaking community of Belgium were excluded from the analysis. This leaves 580 out of a total of 589 Belgian municipalities in the sample. Including the German community in the sample and estimating a separate border effect for this group would increase our number of directional border effects from 9 to 16. At the same time, these additional border effects would be difficult to estimate given the small number of municipalities, the small size of their labour markets and their remoteness from Flanders and Brussels. Moreover, these German municipalities do not constitute a legal geographical entity with the same level of competences as the Walloon, Flemish and Brussels regions.

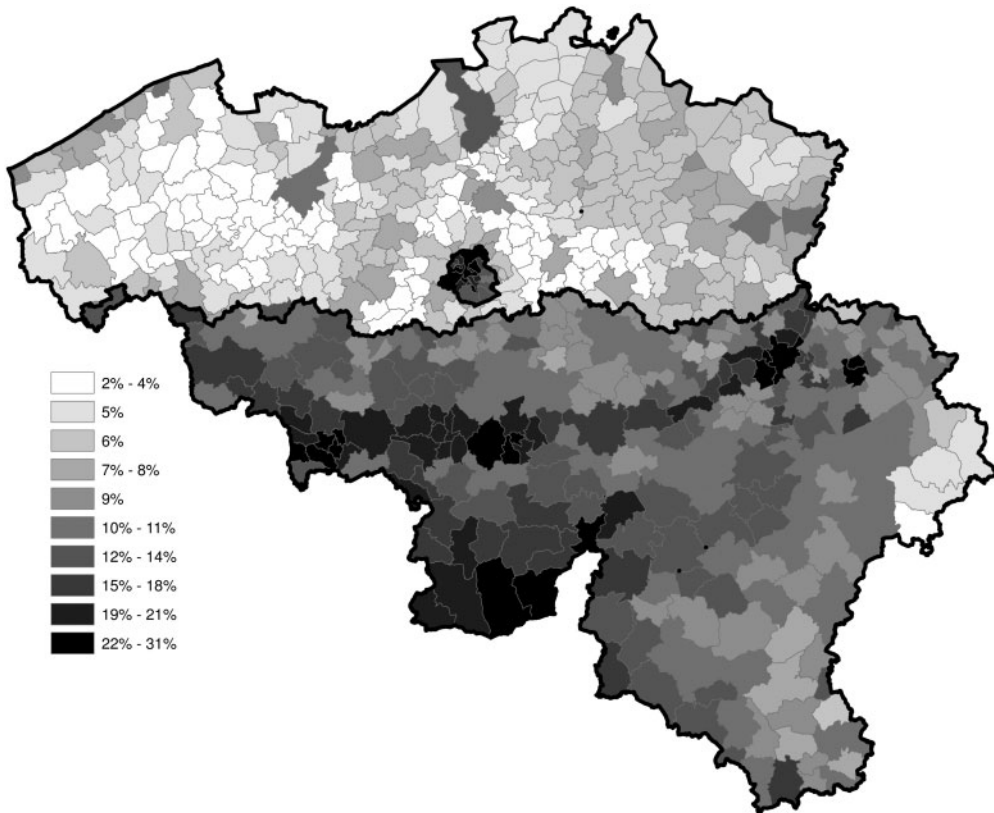


Figure 1. Unemployment rates for Belgian municipalities in 2008.
Source: Steunpunt WSE.

The three Belgian regions are also characterized by strong and persistent differences in economic performance. The capital region of Brussels is unmistakably the centre of Belgian economic activity, hosting 17% of total Belgian payroll employment. Despite being Belgium's most important economic hub, the Brussels unemployment rate is the highest in the country. This can also be seen in Figure 1, which shows unemployment rates for 2008 at the municipal level and illustrates the stark contrast between the labour market performance of Brussels, where unemployment reached 16%, and Flanders, where unemployment was only 3.9%. The Walloon unemployment rate, with 10.1%, was also significantly higher than in Flanders. These regional differences in labour market performance arose in the aftermath of the 1970s oil crises and the decline of traditional steel and coal industries, and have persisted ever since (Torfs, 2008).² It is noteworthy how the linguistic and regional borders in Belgium can be clearly recognized on this map of municipal unemployment rates. Municipalities in Brussels

² Remarkable is also that the exact location of the historically important coal basin in Wallonia can still be clearly recognized, running East–West and parallel to the language border, although the last mine in this area closed in 1984.

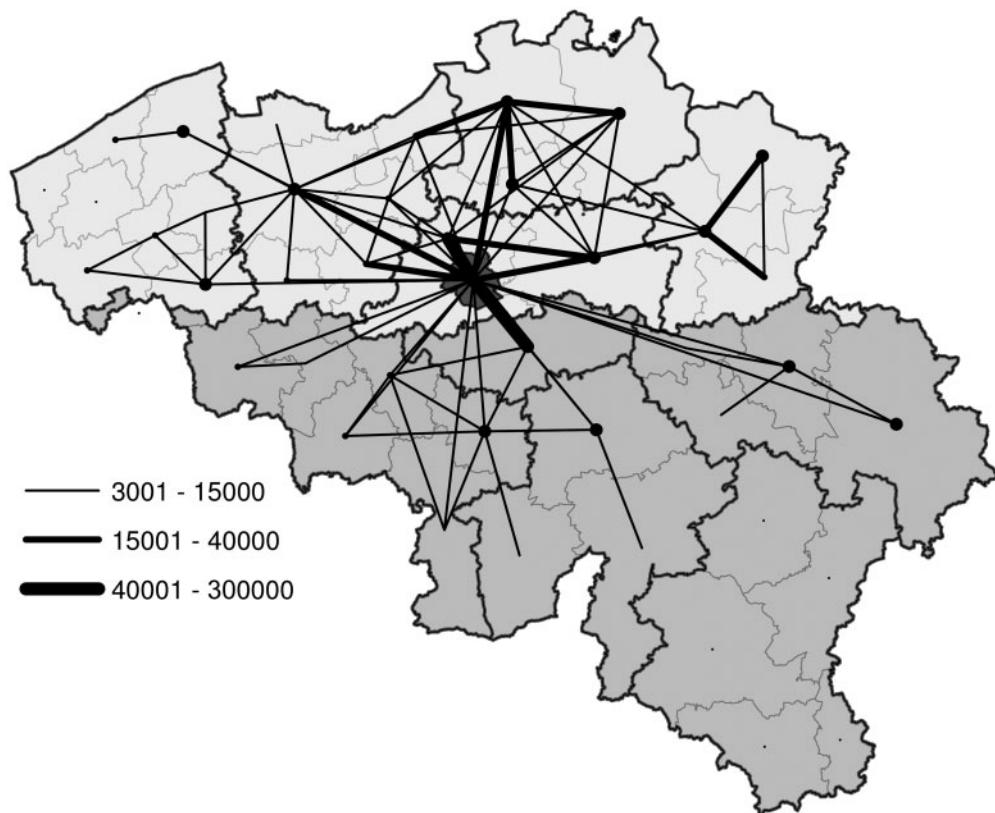


Figure 2. The main commuting flows in Belgium.

and Wallonia have consistently higher unemployment rates compared with their Flemish counterparts located just a few kilometres away.

Figure 2 uncovers the salient spatial patterns of commuting flows in Belgium, aggregating flows at the district level.³ Only inter-district flows containing more than 3000 workers are shown and larger commuting flows are represented by thicker lines. Also here, the role of the central capital region of Brussels as the nation's most important employment centre becomes clear from the web of commuting lines surrounding it. The northern city of Antwerp and the western city of Ghent play an important role for the northern region of Flanders. In the southern region of Wallonia, most commuting takes place between and around the cities of the axis Mons-Charleroi-Liège. Notably, there is not one district-level commuting flow running between the northern region of Flanders and the southern region of Wallonia that contains more than 3000 workers. Considering pairs of municipalities located within a distance from

3 A district or 'arrondissement' is the second smallest level of administrative region in Belgium of which there are 43 in total.

10 to 30 km, the size of a within-region commuter flow is on average 7.5 times larger than a flow of commuters crossing the border separating Flanders from Wallonia. These findings are striking as there are no legal or administrative barriers to labour mobility across regions whatsoever.

The gravity model developed in this article provides a framework to analyse the determinants of the spatial structure of commuter flows that is illustrated in Figure 2. After controlling for factors such as the geographic distribution of workers and jobs, and the travel time by public transport and by car, it is found that regional borders remain a significant hurdle to commuting. Our findings are in line with Falck et al. (2012), who use data on historic language differences between German dialects as a proxy for contemporary cultural differences and find that these form a hurdle to migration flows. This deterrent effect of regional borders on labour mobility offers a possible explanation for the lack of between-region correlation in labour market outcomes as observed by Fuchs-Schündeln and Bartz (2012). Given the large disparities in local labour market performance, our results therefore suggest that a lot can be gained from policies to reduce the deterrent effects of regional borders on labour mobility, such as improving language education or promoting cross-border cultural exchange. The remainder of this article is structured as follows: Section 2 develops a micro-founded gravity equation. Section 3 discusses our estimation strategy. Section 4 introduces the data and discusses the estimation results. Section 5 concludes.

2. A micro-founded gravity equation for commuting

2.1. Deriving a micro-founded gravity equation for commuting

Our gravity equation for commuting builds on Anderson and Van Wincoop (2003), who derive a gravity equation for international trade flows. The labour supply of a locality is assumed to be fixed and workers are residentially immobile. Commuting is the only form of labour mobility available to workers. For the sake of simplicity, assume that each locality hosts a single firm. The firm operating in locality d produces output Y_d using a CES technology with labour differentiated by locality as the sole input.

The assumption that labour is differentiated across localities seems strong. We argue that, apart from offering a convenient functional form, this assumption captures some essential spatial features of the labour market. Labour market agents come in all shapes and forms and not all varieties fit well together: a worker's skill set can differ from an employer's educational requirements, career prospects desired by workers can differ greatly from those on offer and a worker's personality does not necessarily fit a firm's working environment. Preferably, workers and employers would want to avoid having to bear large commuting costs and therefore will search for a suitable match nearby. But labour market heterogeneity decreases the probability of finding the right match locally. Firms and workers could wait until random shocks free up a suitable partner nearby, but will often find that the opportunity costs of waiting outweighs the commuting cost associated with a more distant match. If matches are broken randomly over time and tangible or intangible costs to moving residence are high, we would end up in a situation where firms source

workers from different localities, which is captured by our model. A similar dynamic matching process is described by Hausmann et al. (2013), where firms that locate in a region in which they are an industry pioneer face uncertainty about the relevant characteristics of the local workforce. Consequently, they choose to hire non-local workers that possess the required industry experience. Their empirical findings suggest that these firms do form some suboptimal matches with local workers, but expand their geographic recruiting distance for key-workers. There is a strong analogy with the spatial commuting pattern generated by our model, where in addition to local workers, firms also hire more remotely. The value of these remote workers might simply arise from their availability at the time of the vacancy posting. Rosen (1978) and Dupuy (2012) discuss in greater detail some formal derivations of aggregate CES production function using micro-foundations that rely on worker and job heterogeneity and matching.

More formally, write C_{od} for the amount of labour from locality o used by the firm in locality d . The production function is given by

$$Y_d = \left(\sum_{o=1}^R (A_o C_{od})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where the parameter A_o reflects differences in the productivity of the local workforce. The parameter $\sigma > 1$ is the elasticity of substitution between workers from different localities.

A firm from locality d which minimizes costs conditional on some exogenous output level has the following demand for locality o 's labour:

$$C_{od} = w_{od}^{-\sigma} \left(\frac{1}{A_o \Omega_d} \right)^{1-\sigma} \sum_{o=1}^R w_{od} C_{od}, \quad (2.1)$$

where w_{od} is the wage earned by workers commuting from o to d , and

$$\Omega_d = \left(\sum_{o=1}^R \left(\frac{w_{od}}{A_o} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \quad (2.2)$$

is the wage index faced by firms in d . We will write $B_d \equiv \sum_{o=1}^R w_{od} C_{od}$ for firm d 's total wage bill.

Commuting is costly, and hence a spatial equilibrium where all workers are indifferent to their location of work requires the firm in d to pay a higher wage w_{od} to commuting workers from o , compared to the wage w_o these workers would earn locally. We assume that commuting costs are a fixed proportion of wages and write $\tau_{od} - 1 > 0$ for the commuting cost between o and d as a fraction of w_o . A spatial equilibrium then requires $w_{od} = w_o \tau_{od}$. Note that τ_{od} can be interpreted as an implicit wasteful ad-valorem tax on commuting.

Next, write E_o for the total observed earnings of all workers living in locality o

$$E_o \equiv \sum_{d=1}^R w_{od} C_{od}. \quad (2.3)$$

Substituting Equation (2.1) into (2.3) and using $w_{od} = w_o \tau_{od}$ allows to write local wages w_o as:

$$\left(\frac{w_o}{A_o}\right)^{1-\sigma} = \frac{E_o}{\sum_{d=1}^R \left(\frac{\tau_{od}}{\Omega_d}\right)^{(1-\sigma)} W_d}. \quad (2.4)$$

This in turn can be substituted into Equation (2.1) to get:

$$w_{od} C_{od} = \frac{E_o}{\sum_{d=1}^R \left(\frac{\tau_{od}}{\Omega_d}\right)^{(1-\sigma)} W_d} \left(\frac{\tau_{od}}{\Omega_d}\right)^{1-\sigma} W_d, \quad (2.5)$$

Next, define Y^T as the total wage bill paid (and earned) in the economy, and define $b_d = W_d/Y^T$ and $e_o = E_o/Y^T$, which are the shares of d 's wagebill and o 's earnings, respectively, such that Equation (2.5) becomes:

$$w_{od} C_{od} = \frac{E_o W_d}{Y^T} \left(\frac{\tau_{od}}{\Pi_o \Omega_d}\right)^{1-\sigma}, \quad (2.6)$$

with

$$\Pi_o' \equiv \Pi_o^{(1-\sigma)} = \left(\sum_{d=1}^R \left(\frac{\tau_{od}}{\Omega_d}\right)^{(1-\sigma)} b_d\right). \quad (2.7)$$

After substituting the expression for $(A_o/w_o)^{(1-\sigma)}$ from Equation (2.4) into Equation (2.2), Ω_d can be written as:

$$\Omega_d' \equiv \Omega_d^{(1-\sigma)} = \left(\sum_{o=1}^R \left(\frac{\tau_{od}}{\Pi_o}\right)^{(1-\sigma)} e_o\right) \quad (2.8)$$

Equations (2.6) to (2.8) are the labour market equivalents of the Anderson and Van Wincoop (2003) gravity model for trade flows. To express commuter flows in quantities, rather than monetary flows as customary in the international trade literature, we rewrite Equation (2.6) in terms of number of workers, by using the fact that $w_{od} = w_o \tau_{od}$ and therefore $E_o = \sum_{d=1}^R w_{od} C_{od} = w_o \sum_{d=1}^R \tau_{od} C_{od}$:

$$C_{od} = \frac{\bar{E}_o W_d}{Y^T} \tau_{od}^{-\sigma} \left(\frac{1}{\Pi_o' \Omega_d'}\right) \quad (2.9)$$

where $\bar{E}_o = \sum_{d=1}^R \tau_{od} C_{od}$ is the new adjusted mass variable for the locality of origin.

Equation (2.9) is our final gravity equation, derived from a spatial labour market model. Together with Equations (2.7) and (2.8) it represents the system of equations describing commuting flows. The origin mass variable equals the sum of all bilateral commuter flows originating from that locality, weighing each flow by its bilateral commuting costs. The mass variable of the locality of destination is simply the total wage bill in that locality.

In line with AvW, the gravity equation contains an origin-specific term, Π_o' , and a destination-specific term, Ω_d' . Both depend on all bilateral commuting costs in the economy and on the distribution of economic activity around the origin and destination

locality. These terms are similar to the factors which AvW label ‘multilateral resistance terms’ (or MR-terms) in the context of international trade. As emphasized by AvW, ignoring the MR-terms leads to biased parameter estimates.

2.2. Relation with gravity equations derived from spatial interaction models

Gravity equations have a long tradition in fields other than international trade theory, most notably in the field of spatial interaction modelling. Often, these gravity equations are not formally derived from underlying behavioural assumptions nor theory. International trade has been an exception in this respect. Anderson (1979) provided the theoretical foundations that served as a basis for the development of gravity equations in the international trade literature. This evolution has occurred largely parallel to the developments in spatial interaction modelling. However, there are some striking similarities between the relatively recent AvW model and some older spatial interaction models, more precisely the Wilson doubly constrained model.

A naive gravity equation for commuting could start with the assumption that commuter flow F_{od} between an origin o and destination d can be modelled as a multiplicative function of: (i) the number of workers in the origin o ($N_o = \sum_d F_{od}$), (ii) the number of jobs in the destination d ($J_d = \sum_o F_{od}$) and (iii) a factor (ϕ_{od}) reflecting the effect of distance, often an exponential or power function of geographical distance.

$$F_{od} = N_o J_d \phi_{od} \quad (2.10)$$

This specification suffers from two important problems. First, it only takes into account the characteristics of the origin and destination regions and ignores the influence of third regions on the predicted flow between o and d . Second, doubling the mass variables would quadruple the predicted flows in the system.

The doubly constrained spatial interaction model provides one way of dealing with these concerns by adding two additional terms Q_o and R_d to the gravity equation. These terms constrain the model such that the predicted outflows $N_o = \sum_d F_{od}$ and inflows $J_d = \sum_o F_{od}$ in every locality remain constant. It is straightforward to show that the constraints hold when the gravity equation and the two balancing factors are defined as follows:

$$F_{od} = N_o J_d Q_o R_d \phi_{od} \text{ with } Q_o = \left(\sum_d J_d R_d \phi_{od} \right)^{-1} \text{ and } R_d = \left(\sum_o N_o Q_o \phi_{od} \right)^{-1}. \quad (2.11)$$

Although Q_o and R_d introduce the influence of third regions in the model in an intuitively appealing way, it remains debated whether these factors correctly reflect these spatial dynamics (see, for example, Fotheringham, 1983). In the end, their only aim is to uphold the constraints. This critique applies to our model and the trade-gravity literature as well. There exist interesting other approaches such as the model of Alonso (1978) in which the degree to which the flow totals are constant is flexible, with models (2.10) and (2.11) as two extreme cases. That even Alonso’s model has been largely ignored in the trade-gravity literature (with some exceptions, such as Bröcker, 1989) is particularly remarkable, as it seems realistic that total trade flows are not constant and would increase in response to trade liberalization.

The doubly constrained gravity model can be derived from entropy maximization (or information minimization) as in Wilson (2010). Anas (1983) provides a link between this doubly constrained gravity equation and a discrete choice framework. We will refer to this model as Wilson’s doubly constrained gravity equation (Wilson, 2010). Even

though AvW do not explicitly refer to their model as being constrained, they (and we) implicitly incorporate both a supply and a demand side constraint. A fundamental difference between both models relates to the nature of these constraints: On the supply side, instead of constraining total number of outgoing commuters (the number of resident workers in a locality), we constrain total workers' earnings in each locality to remain fixed. On the demand side, Wilson's model constrains the total number of incoming commuters (number of jobs), whereas we constrain our model such that the outgoing wage payments in each locality (the total wagebill) always equals their initially observed value. These monetary constraints enter our model naturally through the derivation of the labour demand framework by substituting for the local wage levels, w_o . This allows our gravity equation to capture differences in the local average productivity level of workers, A_o . Our approach highlights that a model does not need to constrain the unit of the dependent variable: despite the fact that we consider commuter flows, from an economic perspective, it seems intuitive to keep the total costs (wagebill) of the firm in any destination region fixed when considering changes in commuting costs, rather than the number of in-commuters.

The performance of commuting models is traditionally judged on their ability to replicate the trip distribution matrix, using measures such as the root mean squared error (RMSE) (see Knudsen and Fotheringham, 1986). Although the focus of Wilson's doubly constrained model is on commuting flows and imposes constraints in the unit of the dependent variable (total inflow and outflow of workers), we show that the RMSE and the Akaike Information Criterion (AIC) of our model is actually smaller. This confirms our prior intuition that in this context, the monetary constraints are a sensible choice. In addition to a superior model fit, monetary constraints are more appropriate to control for the role of wages as a spatial equilibrating mechanism between local labour markets, as they take into account how firms value different workers and how local labour productivity differences affect the spatial commuting patterns. Failing to control for the local wage level can lead to biased estimates of the border effect if spatial productivity is not randomly distributed with respect to the border location.

3. Estimation strategy

A log-linearized version of the gravity Equation (2.9) could be estimated by OLS⁴. But as argued by Silva and Tenreyro (2006) this approach is problematic for two reasons: first, in the presence of heteroskedasticity, Jensen's inequality implies that, log-linear transformations will cause the error term to become correlated with the covariates.⁵ Second, by log-transforming Equation (2.9), all observations with a commuter flow equal to zero drop out of the analysis. This is the case for about 65% of all observations in our sample. This type of censoring leads to sample selection bias. To overcome both problems, we treat commuter flows as count data. Count models explicitly allow for

4 For an insightful discussion on the evolution of estimation techniques of gravity equations in the trade literature, see Burger et al. (2009).

5 Flowerdew and Aitkin (1982) describe how the expected value of the logarithm of a random variable depends on its variance. So, in the presence of heteroskedasticity, where the variance of the error term depends on the covariates, its logarithm depends on the regressors, hence violating the consistency condition of OLS, leading to biased estimation. See also Silva and Tenreyro (2006).

zero as a possible (and likely) outcome and do not suffer from bias in the presence of heteroskedasticity. We use a negative binomial model that allows the variation of the count variable to exceed its mean (overdispersion).⁶

Assume that commuting costs are a log-linear function of geographical distance ($dist_{od}$) and a dummy capturing the effect of regional borders (B_{od}), such that

$$\tau_{od} = dist_{od}^{\alpha_1} e^{\alpha_2 B_{od}} \quad \text{or} \quad \ln \tau_{od} = \alpha_1 \ln dist_{od} + \alpha_2 B_{od}. \quad (3.1)$$

For within-locality commuting, the ‘internal distance’ $dist_{ii}$ is assumed to be directly proportional to the square root of the area of each municipality and calculated according to the formula $dist_{ii} = (2/3)\sqrt{area_i/\pi}$, as in Head and Mayer (2000).

The stochastic negative binominal model for the gravity Equation (2.9) is given by:

$$\begin{aligned} C_{od} &\sim \text{Poisson}(\exp(\eta_{od} + v_{od})) \\ e^{v_{od}} &\sim \text{Gamma}(1/\gamma, \gamma) \\ \eta_{od} &= -\ln Y^T + \ln \bar{E}_o + \ln W_d - \sigma\alpha_1 \ln dist_{od} - \sigma\alpha_2 B_{od} \\ &\quad + \ln \Pi'_o + \ln \Omega'_d \end{aligned} \quad (3.2)$$

where γ is the overdispersion parameter, $\bar{E}_o = \sum_{d=1}^R \tau_{od} C_{od}$ and $W_d = \sum_{o=1}^R w_{od} C_{od}$. Equation (3.2) is then estimated and solved subject to the set of non-linear constraints 8 and 7.

To solve this non-linear system of equations, we apply a nonlinear version of the Gauss-Seidel method.⁷ Using some initial guess for the vector Π'_o and the parameters α_1 and α_2 governing the commuting costs, we calculate a first approximation for Ω'_d . Using these values Ω'_d , we in turn calculate an improved guess for Π'_o . Iteration proceeds between updating Ω'_d and Π'_o for given α_1 and α_2 until convergence is achieved. Equation (3.2) then is estimated using maximum likelihood, providing updated values for the commuting costs parameters α_1 and α_2 , after which new values for Ω'_d and Π'_o are iteratively calculated. This entire process is repeated until α_1 and α_2 converge. As σ cannot be identified without knowledge of the specific elements of τ_{od} , an assumption on σ is required to calculate the MR-terms. The analysis proceeds using $\sigma = 2$.⁸ As the MR-terms are stochastic in nature, we use a bootstrap method to calculate standard errors on the coefficients in all tables using 200 replications.

The resulting coefficient on the border dummy $-\sigma\alpha_2$ does not correspond to the percentage change in commuting due to the presence of the border as in a standard regression. The ceteris-paribus condition is violated because other variables in the model change depending on the absence or presence of a border (see AvW and Feenstra, 2004). Values for the MR-terms have to be recalculated to conduct comparative static analyses. As in AvW, we consider only the direct effect of varying the border effect on Π'_o and Ω'_d and ignore changes in the shares e_o and b_d , as well as in the mass variables.

6 The critique of Bosquet and Boulhol (2010) on the use of the negative binomial model does not apply in this context as our dependent variable, the number of commuters, is scale independent.

7 A similar method is also discussed in Head and Mayer (2014). For a more general description of nonlinear Gauss-Seidel methods, see Vrahatis et al. (2003).

8 Varying the value of σ to other (extreme) values of sigma reported in the relevant literature, leaves the results qualitatively unaltered. We refer to Section 4.2 for a discussion of the sensitivity of the border effect to the chosen value of σ .

Define the ‘border effect’, X_{od} , as the percentage difference between a commuter flow C_{od} between two localities o and d which are separated by a border ($B_{od}=1$), and the commuter flow C_{od}^* under the hypothetical scenario in which the effect of a set of borders B_{ij} is set to zero. From Equation (2.9) and (3.1) it follows that

$$X_{od} = \frac{C_{od} - C_{od}^*}{C_{od}^*} = \frac{(\Pi_o' \Omega_d')}{(\Pi_o^* \Omega_d^*)} e^{-\sigma \alpha_2} - 1, \quad (3.3)$$

where Π_o^* and Ω_d^* are the recalculated multilateral resistance terms, setting some $B_{ij}=0$. Obviously, the border effect X_{od} is only defined for localities that are separated by a border ($B_{od}=1$).

The fact that some of the explanatory variables in our estimation equation are not deterministic implies that the regular standard errors of the coefficients may be downward biased. All reported standard errors are therefore bootstrapped using 200 replications, including those of derived statistics such as the border effects reported in Section 4.

4. Estimating the effect of regional borders on commuting

4.1. Data description

Highly disaggregated data on the number of daily commuters between 580 Belgian municipalities⁹ was obtained from the Belgian National Social Security Office (NSSO) for the year 2008. This administrative source covers total Belgian payroll employment, but excludes the self-employed. About 76% of the Belgian payroll workers work in a different municipality than the one they live in. Of them, 16% work in a different NUTS1 region. Our unit of analysis is the number of commuters between pairs of municipalities. Including within-municipality commuting flows, our dataset consists of 336,400 datapoints (containing 3,274,709 workers). Of these pairs 217,721 or about 65% do not have any commuting between them. The largest (great-circle) distance one can travel (280 km) within Belgium’s boundaries is between the sea-side municipality of Koksijde and Aubange, a municipality near the Luxembourg border. The non-zero commuter flow covering the largest great-circle distance runs between the municipalities of Koksijde and Waimes, located 247 km away from each other. Arguably, the relatively small difference between the farthest non-zero commuter flow and the maximum commutable distance implies that all municipality pairs in the sample are within a commutable distance from one another. The vast majority of commutes take place within much smaller distances: the median commuter bridges 9.3 km, the average distance commuted is 21.3 km and about 75% of all commutes take place within 24 km.

To analyse commuter flows, travel time is likely to be more relevant to commuters than simple great-circle distance because it controls for factors such as the quality of transport infrastructure. The analysis includes three different measures of inter-

9 Nine municipalities belonging to the small German speaking community of Belgium were excluded from the analysis. This leaves 580 out of a total of 589 Belgian municipalities in the sample. Estimating a separate border effect for this group would increase our number of directional border effects from 9 to 16. At the same time, these additional border effects would be exceedingly difficult to estimate given the small number of municipalities, the small size of their labour markets and their remoteness from Flanders and Brussels. Moreover, these German municipalities do not constitute a legal geographical entity with the same level of competences as the Walloon, Flemish and Brussels regions.

municipality distance. A first proxy is the geographical distance (dist_{od}) between the town halls of both municipalities. Additionally, we consider travel time by car (car_{od}), obtained through the Google Maps API and travel time by public transport (pubtrans_{od}), obtained from the website of the main Belgian train operator, NMBS.¹⁰

A substantial part (782,927 workers or 23.9%) of Belgian commutes takes place within municipality borders. We proxy intra-municipality travel times using the following methodology: first, a log-log specification is used to regress travel time on distance using data on short-distance intermunicipality commuting. This provides an estimate for the relationship between travel time and distance. Using this relationship, the within-municipality travel times were then predicted, starting from the internal distance measure $\text{dist}_{ii} = (2/3)\sqrt{\text{area}_i/\pi}$. The resulting average within-municipality commuting distance is 2.52 km, with an associated average commuting time of 17.8 min by public transportation or 6.9 min by car. Our results do not change much when we use other proxies for internal distance and travel time or simply exclude within-municipality commuting from the analysis altogether.

The data also contain the average wage paid by the firms in a municipality. We use it to calculate the total wage bill in each municipality, which serves as the mass variable of destination. The origin mass variable, in contrast, has to be calculated iteratively in the estimation procedure as described in Section 3.

4.2. Estimation results, base specification

This section proceeds with the estimation of the gravity equation represented by Equation (3.2). All the empirical specifications include an origin-specific constant term for each of the three regions. This is equivalent to assuming the commuting cost vector contains a region-of-origin specific component. An alternative interpretation follows from the perspective of defining the desired control group, because a separate constant term for each region controls for regional differences in the average size of outgoing inter-municipality commuting flows. By including origin-specific constant terms, we evaluate the size of cross-border commuting flows by comparing them with commuting flows within the same region-of-origin.

We first estimate the border effect using one single dummy variable that indicates whether the commuter flow between two municipality pairs crosses one of the regional borders between Brussels, Flanders and Wallonia. This implies that Table 1 assumes commuting costs to be symmetric for all border crossing. Which border is crossed, or the direction wherein, is not taken into account. This symmetry assumption will be relaxed later on. The border effects are reported in the lower half of Table 1. They are calculated using the comparative static formula [Equation (3.3)], which compares the cross-border flows with all borders intact, relative to the hypothetical cross border flows, where a single border effect is set to zero ($B_{od}=0$) in Π_o^* and Ω_d^* . The border effects in Table 1 differ between border crossings only because of differences in the counterfactual MR-terms. Column (1) of Table 1 shows the result of estimating a specification that corresponds to model (3.2), except for the fact that the coefficients on the variables are not constrained to their theoretical values. Proxies for commuting

10 Public transport times refer to the shortest travel time to get to the destination at 8.30 am on a Tuesday morning, combining all forms of public transport such as train, bus and underground. The data on travel times reflect the situation in June 2011.

Table 1. Estimating the gravity equation for commuting, with a single border-crossing dummy B_{od} indicating any interregional border crossing

	(1)	(2)	(3)
$\ln \bar{E}_o$	1.022*** (0.0267)	1.021*** (0.0174)	1
$\ln W_d$	1.138*** (0.0192)	1.075*** (0.0194)	1
$\ln dist_{od}$	-1.951*** (0.0280)		
$\ln car_{od}$		-2.663*** (0.0989)	-2.555*** (0.117)
$\ln pubtrans_{od}$		-0.530*** (0.107)	-0.679*** (0.132)
B_{od}	-0.941*** (0.0631)	-0.547*** (0.0477)	-0.432*** (0.0412)
$\ln \Pi'_o$	-0.856*** (0.0625)	-0.898*** (0.0367)	-1
$\ln \Omega'_d$	-1.174*** (0.204)	-1.150*** (0.133)	-1
Border effects, X_{od}			
$X_{od}(o \in FL, d \in WL)$	-0.486*** (0.0238)	-0.326*** (0.0232)	-0.300*** (0.0247)
$X_{od}(o \in FL, d \in BR)$	-0.456*** (0.0192)	-0.301*** (0.0221)	-0.278*** (0.0233)
$X_{od}(o \in WL, d \in FL)$	-0.481*** (0.0278)	-0.321*** (0.0260)	-0.296*** (0.0268)
$X_{od}(o \in WL, d \in BR)$	-0.518*** (0.0183)	-0.349*** (0.0235)	-0.323*** (0.0250)
$X_{od}(o \in BR, d \in FL)$	-0.559*** (0.0347)	-0.371*** (0.0312)	-0.343*** (0.0321)
$X_{od}(o \in BR, d \in WL)$	-0.588*** (0.0263)	-0.401*** (0.0267)	-0.371*** (0.0288)
Measures of fit			
RMSE	2474	1434	1023
AIC	2.437	2.346	2.359
N	336,400	336,400	336,400

Bootstrapped standard errors in parentheses * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Column 1 controls for the simple distance measure. Column 2 controls for the two commuting time measures. Column 3 restricts the coefficients on the mass variables and MR-terms to 1 and -1, respectively. The border-specific estimates are calculated using the comparative static formula [Equation (3.3)]. The notation $X_{od}(o \in XX, d \in YY)_{od}$ indicates the effect of the existence of the regional border between regions XX and YY on commuting between them. Note that the border effects differ between border crossings only because of discrepancies in the MR-terms.

costs are the geographic distance between the town halls of both municipalities, $\ln dist_{od}$ and the border dummy, B_{od} . The coefficients on the mass variables are estimated close to unity, as predicted by theory. The coefficients on the MR-terms deviate somewhat further from their predicted value of -1, but are still within a reasonable range. The

Table 2. Results of four separate *t*-tests on the coefficient restrictions imposed by the theory for the estimation results of column 2 in Table 1

H_o	χ^2	<i>p</i> -value
$\beta_{E_o} = 1$	0.08	0.7716
$\beta_{W_d} = 1$	12.53	0.0004
$\beta_{\Pi'_o} = -1$	6.22	0.0126
$\beta_{\Omega'_d} = -1$	0.90	0.3421

effect of distance is clearly negative, as expected. The large and negative coefficient on B_{od} shows that, after controlling for distance and the mass and multilateral resistance of the origin and destination, regional borders act as a barrier to commuters.

Column (2) replaces the simple distance measure by two distance variables which are more relevant to commuters, travel time by car (car_{od}) and public transport ($pubtrans_{od}$). Both variables are included in logs. The time it takes to commute between two municipalities by car is clearly the most important determinant of the two. A 10% increase in travel time by car, reduces the commuter flow by 27%, whereas for travel time by public transport, this is only 5.3%. Comparing the results in column (1) with column (2) reveals that after controlling for the two alternative distance measures, the absolute value of the coefficient on the border dummy decreases. This means that part of the regional border effect captured in column (1) can actually be explained by poor interregional transport infrastructure connecting municipalities across regional borders. In addition, the drop in the associated RMSE shows that the model's fit improves. To make the estimation fully consistent with theory, the coefficient on the mass variables and MR-terms should be equal to 1 and -1 , respectively. Table 2 shows the result of separate *t*-tests on the four coefficient restrictions using the estimates from column (2) of Table 1. It cannot be rejected that the coefficients on the origin's mass variable and the destination's MR-term equal their theoretically consistent values. For the other two coefficients however, this hypothesis is rejected. This is not entirely surprising, given the large sample size. As a robustness check, we solved the model imposing all four restrictions [column (3) of Table 1]. The results remain qualitatively unchanged with a reduction in the estimate of the coefficient on the border dummy from -0.547 to -0.432 .

As in AvW, we need to assume a value for the elasticity of substitution, σ , to solve our model. The results in Table 1 assume an elasticity of substitution (σ) equal to 2. Whether this is a reasonable assumption depends on the nature of the mechanism that is driving the firms' spatial love-of-variety. To the extent that the spatial substitution pattern of workers is driven by differences in the average skill mix of municipalities, a well-chosen value of σ should reflect the degree of substitutability between worker groups with a different educational background. In this context, the relevant literature reports values ranging from 1.1 (see Card and Lemieux, 2001, for men of different schooling types) to as much as 7.5 (see Goos et al., 2010, in the context of task substitutability). Firms' love of variety, as argued in Section 2, could also be driven by the intertemporal dynamics of the labour market, in combination with heterogeneous workers characteristics/vacancy requirements. In this context, we know of no studies

that provide estimates of σ . A sensitivity analysis [based on the unrestricted specification, reported in column (2) of Table 1] shows that our main conclusions hold, regardless of the value of σ .¹¹ Solving the model with $\sigma = 1.1$, the lowest value reported by Card and Lemieux (2001) results in a border coefficient of -0.534 , only marginally lower than -0.547 . Increasing the elasticity of substitution to $\sigma = 7.5$, results in a border coefficient of -0.787 . That the cost of the border increases as the spatial substitutability of workers increases (*ceteris paribus*) is expected. We continue our analysis with a value of 2, a value in the lower range of elasticities reported in the literature, which provides a conservative estimate of the border effect.

4.3. Estimation results, comparison with doubly constrained model

Table 3 shows the results when estimating the classic doubly constrained model from Equation (2.11). The specification is similar to the model presented by Equation (3.2), but instead of constraining the destination's wagebill and the origin's total earnings to their observed values, the total number of jobs and total number of workers are constrained, respectively. Table 3 reports both the unrestricted (column 1) and the restricted (column 2) version of the model. Comparing the unrestricted models [column (2) from Table 1 versus column (1) from Table 3] reveals that the results are quite similar, apart from the coefficient on the border dummy, whose absolute value is estimated substantially higher in Wilson's double-restricted model at 0.802. As argued before, this could be caused by failing to control for the role of wages in the gravity equation: if productivity is not randomly distributed with respect to the location of regional borders, this could lead to biased estimation of the border effect. Tables 1 and 3 also report the RMSE and the AIC criterion to assess the fit of the models. The RMSE reported for the restricted Wilson model (3346) is higher than the one for the restricted specification in column (3) of Table 1 (1023), which suggests that the CES-based model is better at predicting the observed commuting flows. This conclusion carries over to the unrestricted models [column (3) of Table 1 versus column (2) of Table 3]. Also using the Akaike information criterion, the specifications in Table 1 are preferred over the classic doubly constrained model, although for the unrestricted version the difference between both models is minor.

4.4. Estimation results, relaxing assumptions on commuting costs

Gravity Equations (2.1), (2.2) and (2.3) in Table 1 are similar to those commonly used in the context of international trade. We will now alter these specifications to better match the specific features of a typical labour market.

Previously, the border effect was assumed to be homogeneous: it was equal for all regional borders and independent from the direction in which those borders were crossed. This assumption is untenable in the context of interregional commuting. As an example, regional asymmetries in the knowledge of the country's other official language would lead to asymmetries in the effect of the different border crossings. We therefore replace the single border dummy with six border indicators, one for each of the possible border crossings between the three NUTS-1 level regions in Belgium. There might also be omitted region-specific factors which affect commuting behaviour, such as regional

11 This is in line with the findings of Anderson and Van Wincoop (2003).

Table 3. Wilson’s doubly constrained model described by Equation (2.11)

	(1)	(2)
$\ln N_o$	1.001*** (0.0260)	1
$\ln J_d$	1.003*** (0.0151)	1
$\ln car_{od}$	−2.465*** (0.0916)	−2.438*** (0.124)
$\ln pubtrans_{od}$	−0.559*** (0.102)	−0.732*** (0.135)
B_{od}	−0.802*** (0.0532)	−0.827*** (0.0677)
$\ln Q_o$	0.659*** (0.0394)	1
$\ln O_d$	0.705*** (0.0632)	1
Measures of fit		
RMSE	1720	3346
AIC	2.365	2.426
N	336400	336400

Bootstrapped standard errors in parentheses * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Column 1: unrestricted estimation. Column 2: restricted estimation. The balancing factors Q_o
and R_d correspond to AvW’s multilateral resistance terms Π'_o and Ω'_d .

culture, policy or differential preferences of commuters regarding modes of transportation. Failing to control for such factors will lead the coefficients on the directional border crossing dummies to be biased as they will pick up this region-specific distance-decay heterogeneity (see also Melo et al., 2011; Fotheringham, 1983, for further discussion on spatial variation in the distance decay parameter). We therefore introduce a second element of heterogeneity in the commuting cost vector and allow the effect of travel time by car and train to differ between regions. Table 4 presents the estimation results.

Column (1) again shows a specification where the coefficients on the mass and MR variables are not constrained to their theoretically consistent value. Colum (2) imposes these restrictions. All six border effects remain qualitatively unchanged after imposing the restrictions. The results of the formal tests of these restrictions are reported in Table 5 and are similar to our findings in Table 2.

The bottom part of Table 4 reports the comparative statics [using Equation (3.3)]. As before, we hypothetically remove each individual border separately. The result reported for the Brussels–Flanders border crossing therefore corresponds to the percentage change in commuting across the Brussels–Flanders border in the case where only this specific border would be eliminated, but all other borders would still be intact. As expected, allowing for differential coefficients on the respective border crossings increases the discrepancies between the border effects substantially.

The results reveal that the averages reported in Table 1 were masking the presence of both negative and positive border effects. Three of the border crossings turn out to have a positive effect on interregional commuter flows, but they are only significantly

Table 4. Estimating the gravity equation for commuting, with a separate directional border-crossing dummy for each regional-border crossing

	(1)	(2)
$\ln \bar{E}_o$	1.024*** (0.0152)	1
$\ln W_d$	1.037*** (0.0158)	1
$\ln car_{od} \times I(o \in BR)$	-2.296*** (0.179)	-2.196*** (0.197)
$\ln car_{od} \times I(o \in FL)$	-2.799*** (0.103)	-2.746*** (0.115)
$\ln car_{od} \times I(o \in WL)$	-2.542*** (0.0826)	-2.464*** (0.0895)
$\ln pubtrans_{od} \times I(o \in BR)$	-0.471* (0.220)	-0.561* (0.240)
$\ln pubtrans_{od} \times I(o \in FL)$	-0.375*** (0.104)	-0.436*** (0.114)
$\ln pubtrans_{od} \times I(o \in WL)$	-0.523*** (0.0824)	-0.627*** (0.0909)
$\ln \Pi'_o$	-0.847*** (0.0287)	-1
$\ln \Omega'_d$	-1.107*** (0.0890)	-1
Estimates of the coefficients on the border dummies, B_{od}		
$B_{od}(o \in FL, d \in WL)$	-0.472*** (0.124)	-0.462*** (0.110)
$B_{od}(o \in FL, d \in BR)$	0.306* (0.133)	0.283** (0.0910)
$B_{od}(o \in WL, d \in FL)$	-1.082*** (0.109)	-1.035*** (0.0950)
$B_{od}(o \in WL, d \in BR)$	0.319* (0.155)	0.304** (0.102)
$B_{od}(o \in BR, d \in FL)$	-0.640* (0.274)	-0.621* (0.264)
$B_{od}(o \in BR, d \in WL)$	0.245 (0.273)	0.238 (0.247)
Border effects, X_{od}		
$X_{od}(o \in FL, d \in WL)$	-0.294*** (0.0637)	-0.289*** (0.0577)
$X_{od}(o \in FL, d \in BR)$	0.229* (0.108)	0.210** (0.0740)
$X_{od}(o \in WL, d \in FL)$	-0.540*** (0.0336)	-0.524*** (0.0322)
$X_{od}(o \in WL, d \in BR)$	0.283 (0.156)	0.268** (0.102)
$X_{od}(o \in BR, d \in FL)$	-0.401** (0.145)	-0.392** (0.140)
$X_{od}(o \in BR, d \in WL)$	0.245 (0.360)	0.238 (0.318)
Measures of fit		
RMSE	574	458
AIC	2.320	2.320
N	336,400	336,400

Bootstrapped standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Column 1: unrestricted estimation. Column 2: restricted estimation. The dummy variables $B_{od}(o \in xx, d \in yy)$ indicate the respective directional interregional border crossings, where BR: Brussels, FL: Flanders and WL: Wallonia. $X_{od}(o \in xx, d \in yy)$ shows the effect of the border on commuting for commuters living in region xx and working in yy, where only the dummy of the border crossing between xx and yy is set to zero [calculated according to Equation (3.3)].

Table 5. Results of the tests on the coefficient restrictions imposed by the theory. The tests are based on the results of column 2 (Table 4)

H_o	χ^2	p -value
$\beta_{E_o} = 1$	0.34	0.5585
$\beta_{W_d} = 1$	3.46	0.0628
$\beta_{\Pi'_o} = -1$	21.48	0.0000
$\beta_{\Omega'_d} = -1$	0.76	0.3827

Table 6. Language knowledge in Belgium

	Brussels	Flanders	Wallonia
French	75	95	100
Dutch	59	100	19
French and Dutch	51	57	17

Values are expressed in percentage unless specified.

Source: Ginsburgh and Weber, 2006

different from zero for the flows with destination Brussels. Instead of being deterred, the commuters are actually attracted by the Brussels Capital region. This positive border effect is likely to be caused by the special capital status of the Brussels region. As it is the public administrative centre of Belgium, Brussels hosts a great deal of Belgian public employment: not only the federal administrative institutions are located there, but also the Flemish public administration is headquartered on Brussels territory. Arguably, there could also be a prestige premium attached to working (or running a business) in the capital region. In addition, the cultural divide between Brussels and the other two regions is less likely to be an obstacle for incoming commuter flows, as its capital status and history causes the inhabitants of both regions to feel connected to Brussels. Linguistic differences are also less of a concern for this border crossing, because the bilingual status of Brussels implies both Dutch and French speakers have many opportunities on the Brussels labour market. This special role for Brussels in the Belgian interregional commuting flows was already visible in Figure 2. Another possible cause of this positive border effect would be a higher cost of living (including housing) in a city, forcing workers to relocate in neighbouring regions and commute. Unfortunately our current framework does not allow considering residence choice. Although we believe omitted housing prices may be important in explaining the positive border effects towards Brussels, this is much less likely the case for the other border crossings.

The positive border effect (24.8%) of the Brussels–Wallonia border might appear to be contra-intuitive. Although the effect is insignificant, it suggests that the commuter flow running from Brussels to Wallonia is found to be larger than what would be expected based on observables such as distance or commuting time. A possible explanation is the peculiar geography of Brussels as a predominantly French-speaking

enclave within Flanders territory with workers from Brussels, predominantly French-speaking, having difficulties accessing jobs requiring knowledge of Dutch in the surrounding Flemish municipalities.

The remaining three borders are negative and were driving the negative homogeneous border effect. The border crossing from Wallonia to Flanders exerts the largest negative effect and reduces commuter flows by 53.5%. The reverse border crossing is somewhat smaller and amounts to -29.3% . Also the Brussels–Flanders border crossing reduces commuter flows, by 39.7%. Probable causes of these negative border effects are deficiencies in language knowledge, the extent of which differs between regions. Table 6 provides some insight into the language factor (see Ginsburgh and Weber, 2006). The data confirm indeed that the knowledge of the second country language may be driving the differential border effects. About 19% of Walloons consider themselves proficient Dutch speakers, whereas the percentage of Flemish who considers themselves proficient French speakers is 59%. Ironically, the survey reveals that the percentage of bilingual speakers is higher in Flanders than in the officially bilingual region of Brussels.

5. Summary and conclusion

In this article, we derive a gravity equation for commuting from a simple spatial labour market model and use it to identify the deterrent effect of regional borders on commuting flows. The model assumes that firms produce with a love of variety for workers from different locations. We see our model as a reduced form of more complex labour market models with heterogeneous labour markets. Our approach builds on the work of Anderson and Van Wincoop (2003), who propose a similar model to explain international trade flows. The development of gravity equations in the trade literature has taken place largely parallel to the development of gravity equations in spatial interaction modelling. Interestingly, our gravity equation, based on the AvW trade model, shows some important similarities with the doubly constrained gravity equation, a workhorse model developed by Wilson (2010). We use this model as a benchmark to test the performance of our gravity equation and show that our approach is superior in terms of predictive power (as measured by the RMSE) and fit (AIC). We also argue that the control variables derived from our labour demand model are more appropriate in light of the identification of the border effect.

We use the model to estimate the border effect by applying a negative binomial regression technique on a Belgian dataset containing commuter flows between 580 Belgian municipalities. Belgium is an interesting country for the study of regional borders and their effect on commuting, as the country is multi-regional and multi-lingual, and even a casual look at the pattern of commuting flows reveals interesting regional patterns.

The model is first estimated assuming symmetric commuting costs. After controlling for differences in local economic conditions and multilateral resistances, we find a significant and large deterrent effect of regional borders on the size of inter-municipality commuting flows. We show that failing to control for transport infrastructure leads to an overestimation of the border effect. In a next step, we allow for asymmetric border effects by taking into account the direction in which a border is crossed. The analysis reveals the border effect is highly asymmetric, as it depends on which border is crossed, and even in which direction.

Our results show that regional borders act as a strong barrier to commuters and therefore impose strong spatial imperfections in the labour market. Hence, reducing them could lead to significant welfare gains, in particular in depressed localities located close to potential employment opportunities in a neighbouring region. Only a small fraction of these effects are explained by a lack of interregional transportation infrastructure. The asymmetry of the border effect strongly suggests that region-specific policies that encourage interregional commuting could reap the benefits associated with regionally integrated labour markets. To unveil and quantify the extent to which different possible causes contribute to the border effects, and their regional differences, has to be left as an interesting venue for further research.

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